A SIMPLE ANALYTICAL APPROACH TO DEAL WITH UNOBSERVED FEEDING IN LIFETIME MEASUREMENTS USING A PLUNGER METHOD

by

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Scientific paper
https://doi.org/10.2298/NTRP2304253M

Determination of lifetime of low-lying states using a plunger method could be a subject of systematic uncertainty if, among else, correction of unobserved feeding is not properly taken into account. In this paper, a simple analytical approach is proposed to deduce a lifetime of the state of interest if fed directly by observed and unobserved feedings. Evolution of population of the state of interest for 10 target-degrader distances was simulated, using Monte Carlo approach, assuming a decay scheme of three populated states (a state of interest, one observed and one unobserved state). The simulation was performed using a wide range of initial conditions (lifetimes and detection efficiencies of the three states). A fit on simulated evolution of investigated state using analytical solution of described model (by fixing known parameters from direct, observed feeding: initial population and lifetime) successfully reproduced desired lifetime. In order to successfully apply the method, it is necessary to have experimentally available intensities of unshifted components of peak from at least five target-degrader distances.

Key words: lifetime measurement, unobserved feeding, simulation

INTRODUCTION

Measurements of reduced transition probabilities are important for understanding nuclear structure as they are sensitive on nuclear wave function of the initial and final state of the observed transition. They can be used, among else, to determine electric quadrupole moment, to deduce nature of the state (single-particle versus collective modes), and determine shape coexistence [1, 2].

Reduced transition probabilities could be measured directly by Coulomb excitation measurement or extracted from the lifetime measurements. There are numerous techniques developed depending on lifetime of an investigated state. Commonly used method to determine lifetime of the low-lying states, that is typically of the order of ps, is recoil distance Doppler shift method (RDDS) or colloquially a plunger method [3].

The RDDS method is performed using a plunger device that consists of target and degrader foil that could be placed at various distances from each other. The role of degrader foil is to stop or slow down recoil nucleus, which can be produced by various reactions, such as Coulomb excitation, multi-nucleon transfer, deep-inelastic scattering *etc*. Depending whether γ-rays are emitted before or after the degrader, they will be differently Doppler-shifted which will result in two γ peaks for each transition. Transition occurred in the degrader (in the case of stopping foil) or, after the degrader is called unshifted component, as its peak would be at proper energy since the recoil is in rest or it is properly Doppler-corrected by recoil velocity measured in spectrometer placed after the plunger device. The component emitted before degrader is named shifted component. Velocity of recoil nuclei is of the order of few percentages to a several tens of percentages of the speed of light and target-degrader distances, set to be of the order of time of flight necessary for the recoil to reach the degrader after leaving the target, have values typically from several micrometers to several millimeters. Thus, by changing target-degrader distances, observed number of de-population of excited state will change as well. From the intensity ratios of these two components as a function of target-degrader distance it is possible to deduce the lifetime. The details of the method could be found elsewhere [3, 4].

In general, there is a good agreement between RDDS and Coulomb excitation measurements. However, an interesting case of ^{72,74}Zn isotopes appeared in

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recent years, with a systematic discrepancy between B(E2;$4^+ \to 2^+$) values obtained using Coulomb excitation and RDDS method [5-10].

In all the RDDS measurements considered [5-7], only γ singles spectra were used in the analysis, thus susceptible to the same potential systematic errors that could arise due to inability to separate peaks from different transitions, unknown feeding patterns or unobserved feeding transitions which could lead to apparent increase of the measured lifetimes. Researchers have applied different approaches to solve the problem of unobserved feeding. One approach would be to include the unobserved feeding via one level approximating all possible unobserved feedings coming from continuum or other discrete levels [4]. Its intensity corresponds to the relative intensity ratio of observed transitions populating and depopulating the state of interest, while its effective lifetime can be assumed to be similar as that of the observed feeding [11, 12]. In cases where such assumption is not applicable, wrong feeding estimates would be observed by deviations of the lifetime from the constant value for different target-degrader distances, as explained in [4]. The influence of the unobserved feeding on the lifetime of interest can, for example, be deduced by nonlinear least-square minimization method, as explained in [13].

An attempt to explain a longer lifetime of the 4^+ state in ^{74}Zn deduced by RDDS method, compared to what Coulomb excitation result suggests [8,10], by addition of the unobserved feeding was made in [5], assuming a long lived, but weakly populated state feeding the 4^+ state, which was added to the measured uncertainty. However, it does not explain the observed discrepancy in ^{74}Zn, even though in cases where the unobserved feeding is of low relative intensity and similar lifetime as the observed feeding, its influence should fall within the experimental errors [12].

These kind of discrepancies could be resolved by using a γ-γ coincidence measurement, however it is not always feasible to achieve it. In this paper, authors are trying to deduce lifetime of the state of interest by taking into consideration unobserved feeding. A simple analytical approach based on solution of sequential decay is proposed and validated using Monte Carlo simulations.

METHODS

Bateman equations represent a set of differential equations whose solution provide abundances and activities of nuclei in decay chain as a function of time (*i.e.*, set of decays from one unstable nuclei to another one until reaching stable nuclei, or a sequence of de-excitations of nuclei from higher state to the ground state). In the lifetime measurements of excited states, instead of decay chain, rather particular state of interest could be fed by one or more feeding excited states. In the case of system with N excited states, among which N-1 states directly feed the state of interest (denoted as state 1) a set of N differential equations can be written

$$N_1 = N_{10}\mathrm{e}^{-\lambda_1 t} - \sum_i \frac{N_{i0}\lambda_i}{\lambda_i - \lambda_1}(\mathrm{e}^{-\lambda_1 t} - \mathrm{e}^{-\lambda_i t}) \tag{3}$$

while time evolution of number of nuclei in N_1 state of interest is described as

$$\frac{\mathrm{d}N_1}{\mathrm{d}t} = -\lambda_1 N_1 + \sum_i \lambda_i N_i \tag{4}$$

where N_1 and N_i denote the population of the state of interest and its direct feeders in i states, respectively, as a function of time, while λ_1 and λ_i are the decay constants of those states. Corresponding lifetimes are $\tau_1 = 1/\lambda_1$ and $\tau_i = 1/\lambda_i$.

Solution of the system of first order non-homogeneous linear differential equation described with eqs. (1) and (2) can be easily generalized from the system of two equations into

$$N_1 = N_{10}\mathrm{e}^{-\lambda_1 t} - \sum_i \frac{N_{i0}\lambda_i}{\lambda_i - \lambda_1}(\mathrm{e}^{-\lambda_1 t} - \mathrm{e}^{-\lambda_i t}) \tag{5}$$

where N_{10} and N_{i0} are the initial numbers of particles in the state of interest and state i, respectively.

For the purpose of finding lifetime of the state of interest, decay analysis of only three states has been considered, as shown in fig. 1.

The first state, with unknown life time τ_1 (aiming to deduce) and unknown initial population N_{10}. The second state corresponds to the observed state that directly feeds the state of interest, with known lifetime τ_2 and population N_{20} (*i.e.* as it is observed feeding, the values are deduced from the experiment directly). The

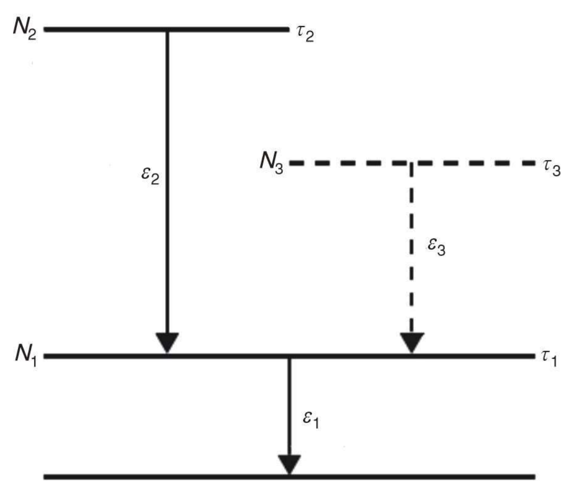

Figure 1. A schematic representation of decay scheme used to deduce lifetime of state 1: N_1 and τ_1 – population and lifetime of state of interest; N_2, τ_2 and N_3, τ_3 are populations and lifetimes of direct observed and unobserved feeder of state of interest, respectively, while ε_1, ε_2, and ε_3 represent absolute detection efficiencies for corresponding transitions

feeding pattern of the second state does not need to be considered, as only its effective lifetime and population, by which it feeds investigated state of interest, are needed. Furthermore, there could be several observed direct feedings, but again, population of those states and lifetimes would be measured and could be directly included in eq. (3). Therefore, without a lack in generality, it is enough to assume only one state that directly feeds the state of interest. Finally, the third state corresponds to unobserved feeder of the state of interest (therefore with unknown lifetime τ_3 and population N_{30}). Similarly, there could be several unobserved feeders, but what is considered in the simulation, and would be considered in experimental analysis, is effective feeding of state of interest.

The evolution of population of the state of interest is simulated for 10 target-degrader distances up to 500 μm. Recoil velocity of 20 μm(ps)$^{-1}$ was assumed, which is within the range of velocities typical for plunger experiments, and for the largest target-degrader distance, recoil nuclei time of flight is 25 ps. At each (infinitesimal) step of time, it was sampled whether decay from any of three states has occurred and at the same time it was sampled whether transition from the corresponding state has been detected or not. Sampling was performed using Monte Carlo method. Number of detected transitions was integrated for the period that corresponds to the time of flight between each target-degrader distance. The process was replayed for each state and target-degrader distance. As an output, a number of survived nuclei in state 1, for each target-degrader distance is given. In the plunger experiment, it would correspond to the measured intensity of unshifted component of transition emitted after the degrader. Simulated results were then fitted to eq. (3) by fixing parameters N_{20} and τ_2 (being known) and leavingN_{10}, τ_1, N_{30} and τ_3 as free parameters, from which the lifetime of the state of interest τ_1 was extracted.

RESULTS

Monte Carlo simulation of the system, illustrated in fig. 1, was performed for numerous combinations of the three states in order to cover different possible experimental situations when τ_1 is shorter or longer compared to τ_2. Lifetimes of the unknown feeding τ_3 were also covering a large range of lifetimes, from several lifetimes shorter to several lifetimes longer than the lifetime of interest τ_1. It was assumed that each transition could have different energy and thus, its detection efficiency ε would be different. For each simulated combination of lifetimes, an integer value of detection efficiency ε was uniformly sampled between 6 % and 16 %, thus, within the range of absolute efficiencies found in contemporary γ -ray spectrometer systems. Initial number of populated states were 10^6, $5\cdot10^5$ and $3\cdot10^5$ for N_{10}, N_{20}, and N_{30}, respectively. Values N_1, N_2, and N_3 would experimentally correspond to the number of detected γ transitions (the peak area times efficiency).The N_{30} as 30 % of intensity of N_{10} is set as an upper limit of population since it is expected that such a high intensity of transition should be observed in γ-ray spectrum. Smaller intensities would make smaller influence on the evolution of N_1 and therefore N_{30} was not changing during the simulations. Fitted lifetimes of the state of interest, for selected combinations of lifetimes and efficiencies are presented in tab. 1.

In figs. 2 to 4 the simulated evolution of population of the state of interest for 10 target-degrader distances for different initial conditions is presented. Uncertainty of the simulated points was taken into account and expressed as the square root of N_1, for each target-degrader distance, however it is not visible in the figures. Fit of simulated data using eq. (3) is indicated as well.

Results of the deduced lifetimes obtained from the fit of simulated data for different initial conditions, presented in tab. 1. and figs. 2-4, well reproduce initially set value. Therefore, it is possible to apply this approach if there is an indication of unobserved feeding of the investigated state. The only limitation is when lifetimes of the state of interest and feeder state are the same, $\tau_1 = \tau_2$ or $\tau_1 = \tau_3$ (left side of fig. 3), as already mentioned [14]. Although the lifetime for that case is in fairly agreement with the initial value, overall fit did not converge and therefore, that result could not be considered reliable.

CONCLUSIONS

The evolution of intensity of the state of interest that has been fed by two states (one direct observed feeding, and another unobserved) as a function of tar-

Table 1. Fitted lifetimes of the state of interest τ_1 obtained for different combinations of lifetimes (τ_1, τ_2, τ_3) and detection efficiencies (ε_1, ε_2, ε_3)

Lifetimes [ps]			Efficiency [%]			Fitted lifetime [ps]
τ_1	τ_2	τ_3	ε_1	ε_2	ε_3	τ_1
3	15	2	12	7	15	2.917 ± 0.023
3	15	10	11	15	6	2.931 ± 0.051
3	15	20	13	9	7	2.913 ± 0.028
5	7	2	7	13	9	4.871 ± 0.033
5	7	5	14	15	6	4.5 ± 9.9
5	7	20	12	10	8	4.85 ± 0.31
10	5	3	13	15	10	10.06 ± 0.43
10	5	5	11	10	7	9.91 ± 0.20
10	5	15	12	10	8	9.93 ± 0.55
10	5	50	15	7	15	9.9 ± 2.1
20	3	3	13	15	15	19.72 ± 0.39
20	3	10	10	14	6	18.5 ± 7.2
20	3	25	13	9	10	20.9 ± 2.9
20	3	60	16	13	8	20.0 ± 3.1

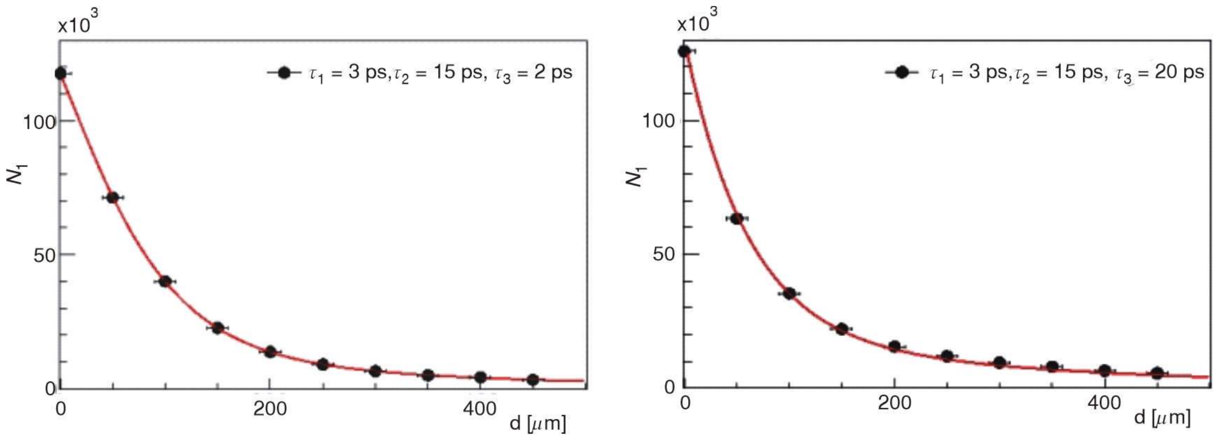

Figure 2. Simulated evolution of intensity of N_1 for different initial conditions; (left side: τ_1 = 3 ps, τ_2 = 15 ps, τ_3 = 2 ps, ε_1 = 12 %, ε_2 = 7 %, ε_3 = 15 %) and (right side: τ_1 = 3 ps, τ_1 = 15 ps, τ_1 = 20 ps, ε_1 = 13 %, ε_2 = 9 %, ε_3 = 7 %), Deduced lifetimes are 2.917 ± 0.023 ps and 2.913 ± 0.028 ps, for conditions presented on the left and the right side of figure

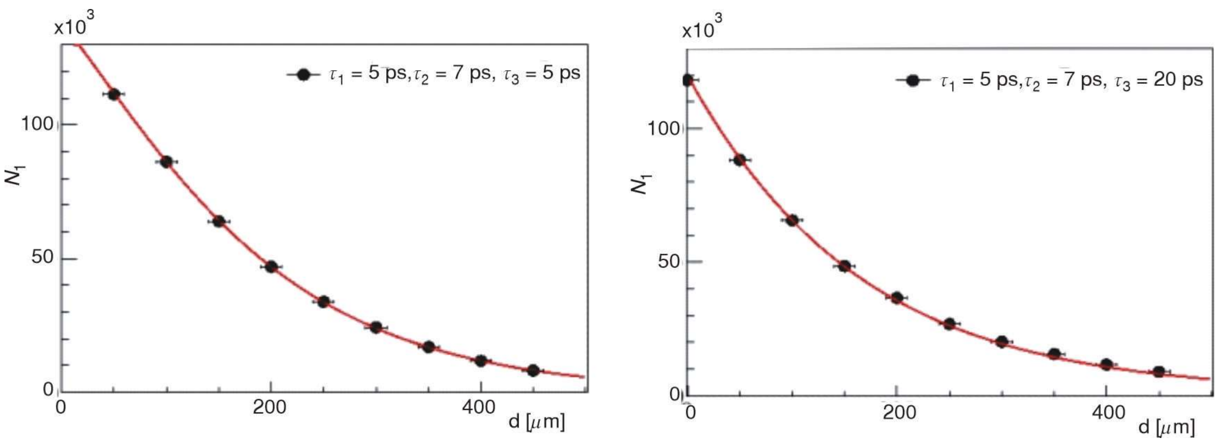

Figure 3. Simulated evolution of intensity of N_1 for different initial conditions; (left side: τ_1 = 5 ps, τ_2 = 7 ps, τ_3 = 5 ps, ε_1 = 14 %, ε_2 = 15 %, ε_3 = 6 %) and (right side: τ_1 = 5 ps, τ_1 = 7 ps, τ_1 = 20 ps, ε_1 = 12 %, ε_2 = 10 %, ε_3 = 8 %), Deduced lifetimes are 4.5 ± 9.9 ps and 4.85 ± 0.31 ps, for conditions presented on the left and the right side of figure

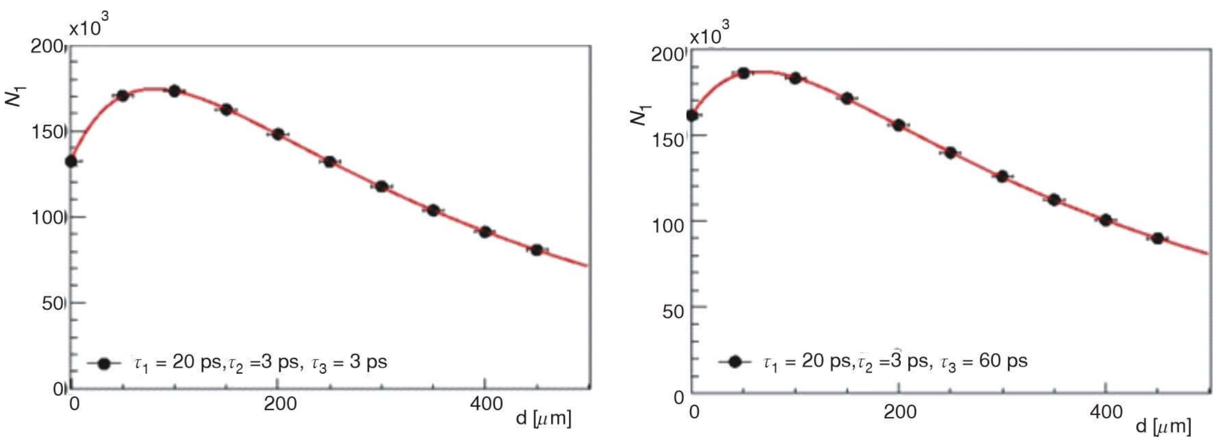

Figure 4. Simulated evolution of intensity of N_1 for different initial conditions; (left side: τ_1 = 20 ps, τ_2 = 3 ps, τ_3 = 3 ps, ε_1 = 13 %, ε_2 = 15 %, ε_3 =15 %) and (right side: τ_1 = 20 ps, τ_1 = 3 ps, τ_1 = 60 ps, ε_1 = 16 %, ε_2 = 13 %, ε_3 = 8%), Deduced lifetimes are 19.72 ± 0.39 ps and 20.0 ± 3.1 ps, for conditions presented on the left and the right side of figure

get-degrader distance was simulated using Monte-Carlo approach. Initial number of nuclei in the state two and its lifetime, N_{20} and τ_2 can be deduced experimentally and should be used as an input parameter in data fit using eq. (3). From the intensity of unshifted component, a lifetime of the state of interest was successfully deduced for a wide range of initial conditions (lifetimes and efficiencies of three simulated states).

This simple method has shown that it is possible to correctly deduce the lifetime of a certain state even when that state is fed by an unobserved transition.

However, as there are four unknown parameters in the model fit, it is necessary to have experimental data from at least five different target-degrader distances, compared to regular RDDS method in which just two target-degrader distances or, even one in the range of sensitivity, can be enough to determine the lifetime of interest [4]. Since the method is sensitive to initial conditions, it is important to have a proper estimation of direct feeder's intensity and lifetime.

ACKNOWLEDGMENT

The authors acknowledge the support of the Ministry of Science, Technological Development and Innovation of Serbia under contract No. 451-03-66/ 2024-03/ 200017.

AUTHORS' CONTRIBUTIONS

Both authors contributed on initial idea and study design. I. T Čeliković, wrote initial code. Both authors were running simulations. T. Milanović has drafted the first version of the manuscript and prepared figures. Both authors made revisions of the draft and approved the final version of the manuscript.

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Received on February 23, 2024
Accepted on March 5, 2024

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ЈЕДНОСТАВАН АНАЛИТИЧКИ ПРИСТУП ПОСТУПАЊА СА НЕУОЧЕНИМ ПРЕЛАЗИМА ПРИ МЕРЕЊИМА ВРЕМЕНА ЖИВОТА ПРИМЕНОМ ПЛАНЦЕР МЕТОДЕ

Одређивање времена живота ниско-побуђених стања користећи планцер метод, подложно је систематским несигурностима ако, између осталог, корекција на пуњење стања неуоченим прелазима није узета у обзир на одговарајући начин. У овом раду, предложен је једноставан аналитички приступ за одређивање времена живота стања од интереса ако га пуне и директни мерљиви прелази и они неуочени. Еволуција популације стања од интереса је симулирана за 10 мета-деградер удаљености, помоћу Монте Карло приступа, претпостављајући шему распада од три побуђена стања (стање од интереса, једно експериментално уочено и једно неуочено). Симулација је изведена користећи широк опсег почетних услова (времена живота и ефикасности детекције три посматрана стања). Симулирана еволуција посматраног стања, фитована аналитичким решењем описаног модела (фиксирањем познатих параметара познатог прелаза који пуни посматрани ниво: почетни број честица у побуђеном стању и време живота), успешно репродукује задато време живота. Како би се метод успешно применио, потребно је имати експериментално одређене интензитете непомерених компоненти гама прелаза са барем пет мета-деградер удаљености.

Кључне речи: мерење времена живота, неуочени прелаз, симулација