STUDY OF MATHEMATICAL FUNCTIONS TO FIT DETECTOR EFFICIENCY CURVE IN GAMMA RAY ENERGY RANGE FROM 59.54 keV UP TO 1408.01 keV

by

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Recently, there have been several significant improvements in the area of the radiation detection system and its instruments, especially those using scintillation or semiconductor gamma ray detectors. Scientists and technicians are interested in studying this progress, which can be useful for the detector's operation and its basic properties, such as energy, shape, and efficiency calibration. In this work, an extended study of various mathematical formulas was conducted to obtain the efficiency best-fitting function, that covers the measured values from low to high energy regions. They can be used to represent the efficiency of a high-purity germanium detector in the regions where accuracy and maximum speed in optimizing the calibration process are very important for gamma spectroscopy. Determination of the activity of environmental samples mainly depends on the efficiency calibration curve of the detection system. The gamma ray energy in the range from 59.54 up to 1408.01 keV used in this work was obtained by using a set of standard radioactive gamma ray sources of certified intensity. The current data analysis shows that most of the mathematical formulas, which represent the fitting curve for the detector full-energy peak efficiency, were quite agreeable with the experimental results.

Key words: mathematical formula, full-energy peak efficiency, fitting process, gamma ray detector, radioactive point source

INTRODUCTION

For setups with different "gamma ray source-to-detector" geometry, the experimental calibration method used depends mainly on the obtained calibration values, which can be either listed in data tables or plotted as (ε, E) 2-D-graphs, where ε is the detector efficiency and E_γ is the photon energy. In this method, monoenergetic or multi-energetic standard radioactive sources of known activity are placed sequentially at a constant distance from the detector in question. Besides, sources with a well-defined energy spectrum must be used for the calibration process.

The experimental method is considered to be the simplest and most direct, although it is time-consuming [1, 2]. Although experimental methods can offer good results with very low uncertainties, some limitations can lead to errors in the final choice of the gamma ray detector full-energy peak efficiency [3, 7]. The sources of errors can be from: variation of the gamma emission probability used by different scientists and technicians, the coincidence summing problems, the use of several standard radioactive sources to cover discrete energy regions, which may lead to the appearance of energy gaps, generating huge uncertainties after finishing the interpolations during the curve fitting and considering the intrinsic rather than total efficiency [8, 9].

Modeling the history of a large number of individual photons passing through the detector, from the moment of emission from the source to the point of absorption inside the detector, is considered to be the principle of Monte Carlo methods. The photon can escape the detector or deposit its energy, in whole or in part, in the detector material, taking into account all secondary photons and particles [10-12]. Monte Carlo methods are used to calculate the gamma ray detector full-energy peak efficiency, while obeying a statistical distribution model, according to the given distribution

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function. The use of Monte Carlo simulations is becoming increasingly important for calculating detector efficiency when using unusual radioactive sources [13, 14]. In the past, the Monte Carlo methods were considered because the running time of a computer program is quite long, since these methods follow each photon history step-by-step, the statistical error must be calculated and, to obtain good probabilities, the number of histories must be large enough [15]. Overall, many scientific groups, as mentioned before, use this method to determine the efficiencies and calibrate detectors.

The quality and quantities in gamma ray spectrometry measurements, on the whole, depend mainly on the awareness of knowing the gamma ray detection efficiency for a *source-to-detector* geometry of the set-up. Since the samples usually can be of various types of materials and with different geometry, calibration is not possible for most of the materials and their containers used in the measurements. These procedures are of changeable dependability and need set-up periods, which can be unsuited with the necessities of regular measurements. Moreover, the limited measurement that, in particular, is necessary for measuring environmental samples, deform the areas under the peaks, which leads to incorrect quantitative results.To overcome these two difficulties, a special efficiency transfer method was proposed [16-25]. It is a computational technique designed to offer a useful and appropriate explanation for problems encountered in laboratory measurements. It allows estimating the efficiency of the gamma ray detector without making calibration by the experimentalists. This makes it possible to increase the precision of the results of the quantitative research using gamma spectrometry and keep away from time uncontrollable measurement runs. This method has been built up based on the concept of changing the gamma ray detector efficiency in case of the absence of the radioactive calibration sources-to-detectors set-up, which should be similar to the samples under investigations and analysis. By computation, it is possible to find out the gamma ray detector efficiency, equivalent to samples of non-point shape and/or in special locations as presented by [26-30] where the efficiency transfer method can be extended to (volumetric, bulk) sources, by calculating the solid angle of the setup and the effect of various absorbing materials.

Several scientists have used a simplified direct approach to determine the gamma ray detector efficiency for a radioactive point-like source at any random location from a cylindrical or well-type detector, in addition to extending this approach to estimate the efficiency for disk and volumetric sources such as coaxial or symmetrically perpendicular cylinder and Marinelli beaker. This approach depends on the mathematical expression for the distance traveled by the photon before being recorded inside the detector as a function of the polar and azimuthal angles and the distance from the source to the detector. Besides, carrying out the integration of the approach over limits of the polar and azimuth angles via simple programs using the trapezoidal rule, [29-30]. Within the framework of this approach, the coefficient of photon attenuation by the source container, the materials of the detector end cap, and the interaction with the source material itself were taken into account.

Newly, extensions to gamma-ray spectrometry have been expanded and become functional in various fields, such as astrophysics and medical therapy, in which extremely precise measurements of gamma ray and calibration processes for the detectors are required. This was achieved using the mode of tracking the interaction of gamma quanta inside the semiconductor and scintillation detectors and the energy released in them. Since the measurements are carried out at separate arbitrary energies, the detector full-energy peak efficiency values at the required gamma ray energies must be obtained from a smooth curve drawn throughout the obtained experimental points. In turn, to take full advantage of the precision of the experimental values, several interpolation methods are required that do not violate the accuracy of the basic records. This is achieved by fitting the basic or main data points with different mathematical formulas. Several mathematical formulas, which depend on the interpolation method and the number of parameters, are used in this work to intensively study the detector efficiency curve in gamma ray spectroscopy in the energy range from 59.54 to 1408.01 keV. All results of the mathematical formulas agree satisfactorily with the results obtained from the measured values.

MEASUREMENT CONFIGURATIONS

In gamma-spectroscopy, the use of a germanium detector makes it possible to identify radionuclides present in an environmental sample with high resolution and determine their massic or volumetric activity (for example, $Bqkg^{-1}$ or BqL^{-1}). The scientists and technicians usually need to know the detector's full-energy peak efficiency with good accuracy for any specific source-to-detector configuration of concern. This section describes the requirements to do the calibration process in the gamma-ray spectroscopy field. The Canberra Germanium Detector Company has been working for many years to improve detectors to have better resolution, better peak-to-Compton ratio, and higher efficiency. In this work, we used Canberra's GC1520 coaxial high-purity germanium detector. A typical sectional view of the detector chamber and characterization are shown in fig. 1(a). Multiple width Nuclear Instrument Modules (NIM) were accommodated in a CANBERRA Model 2000 Min Bin/Power Supply Crate, which provides mounting space and power sources for up to 12 standard single-width NIM electronic units. The Model 3106D power supply provided a constant voltage of +4500, which was required as an operating voltage. The Model 2002CSL preamplifier was integrated into the detector itself, and the Model 2026 amplifier was one of the NIM modules providing a 4   s time constant.

The radioactive point sources include four different radionuclides: ^{60}Co, ^{133}Ba, ^{152}Eu, and ^{241}Am. The sources covered energy range from 59.54 up to 1408.01 keV. The activity of the radioactive sources on June 1, 2009, was (212.1   1.5, 275.3   2.8, 290.0   4.0 and 259.0   2.6) kBq, respectively. The sources are made of two fused disk-shaped polyethylene films with a surface density of (21.3   1.8) mgcm^{-2}, holding in their center a weak radioactive material in the form of a small spot with a diameter of no more than 5 mm. For easy, flexible, and safe handling, the polyethylene-coated radioactive source is secured to a circular aluminum ring, the dimensions of which are shown in fig. 1(b). To measure the spectra, we used a special homemade Plexiglass source holder providing a sufficiently wide solid angle, as shown in fig. 1(c). The sample holder is in two pieces; the first is the base of the holder, which is also used to support the second one, shield the top of the detector and protect it from X-rays and beta particles; the second piece, 2.38 mm thick, is used as a holder for radioactive sources, each of which was located at a distance of 507.49 mm from the front end of the detector and was measured separately. This distance ensures obtaining the amplitude spectra of signals without the influence of the coincidence summation effect, pulse pile-up, and dead time of the detector. The detector efficiency values, obtained under these conditions, will be more suitable for testing various fitting functions. The measurements and recording of the signals from the interaction of the gamma rays with the active material of the detector were carried out via the PC USB port using the Canberra Ge-

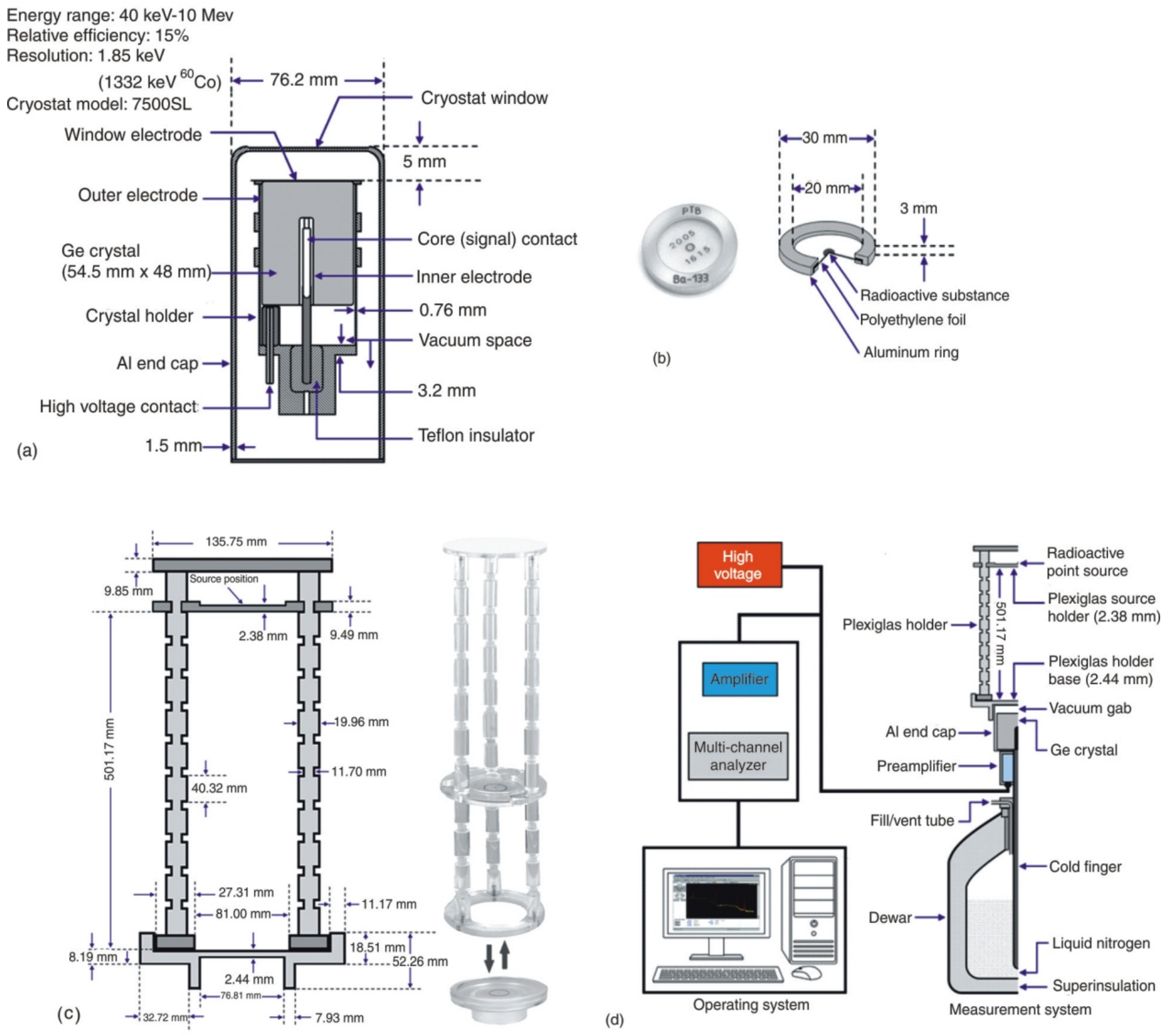

Figure 1. (a) Cross-sectional of the detector, (b) radioactive source dimensions, (c) homemade plexiglass source holder, and (d) experimental set-up

nie 2000-ISO 9001 spectrum acquisition and analysis software. The block scheme of the experimental setup is shown in fig. 1(d). To obtain enough high-count number of events under the interesting peak with 1 % as statistical uncertainty, the measured time was long enough.

Calibration process

To ensure high-quality measurements and calibration, before starting actual measurements with radioactive sources, the detector electronics must be tuned to reduce the effect of any interference that might affect the collected signals. This was done experimentally using a good quality pulse stabilizer, and the setup was located away from the floors and walls of the measurement site. The calibration procedure converts the collected by the II MCA module 8k-channel amplitude spectra of the signals into 8k-channel energy spectra of the gamma rays. Before starting a real measurement process, three radioactive sources, ^{60}Co (1173.23 and 1332.50 keV), ^{137}Cs (661.66 keV), and ^{241}Am (59.54 keV) were used for energy calibration of the gamma ray detector and they roughly covered the entire energy range in recent measurements.

The measured efficiency of the HPGe gamma ray detector varies with the energy of gamma quanta and geometries of the investigated source and detector, therefore, in the quantitative analysis of radioactive environmental samples, standard radioactive sources of the same geometry and gamma-ray energies must be used. The efficiencies for registration of gamma quanta with a certain energy, calculated by eq. 1, can be presented in tables or the form of $\varepsilon(E\ ) = f(E\ )$-graphs

$$\varepsilon(E_\gamma) \quad \frac{N_{\text{net}}(E_\gamma)}{A_{\text{m}} I(E_\gamma) t} \tag{1}$$

where N is the net full-energy (E) absorption peak area (counts), A_{m} – the activity present in the calibration source at the beginning of the measurement, in disintegrations per second (or Bq), $I(E_\gamma)$ – the intensity per decay of gamma ray emission with energy E_γ (the absolute gamma ray emission probability at given energy E_γ, gamma-ray yield, branching ration), t – the measurement live-time (s). The current activity at the time of experiment A_{m} has to be decay corrected using eq. (2)

$$A_m \quad A_{\text{O}} e^{\ \lambda T_d} \tag{2}$$

where A_{O} is the original activity, λ – the decay constant and T_{d} – the decay time. The uncertainty of the measured full-energy peak efficiency of gamma ray detector $\sigma_{\varepsilon(E_\gamma)}$ (variance) is given by

$$\sigma_{\varepsilon(E_\gamma)} \quad \sqrt{S}$$

$$S \quad \frac{\partial\varepsilon(E_\gamma)}{\partial A}^{2} \sigma_A^2 \quad \frac{\partial\varepsilon(E_\gamma)}{\partial I(E_\gamma)}^{2} \sigma_{I(E_\gamma)}^2$$

$$\frac{\partial\varepsilon(E_\gamma)}{\partial N_{\text{net}}(E_\gamma)}^{2} \sigma_{N_{\text{net}}(E_\gamma)}^2 \tag{3}$$

where σ_A, $\sigma_{I(E_\gamma)}$ and $\sigma_{N_{\text{net}}(E_\gamma)}$ are the uncertainties associated with the quantities A, $I(E\ )$, and $N_{\text{net}}(E\ )$ respectively.

From the efficiency values $\varepsilon(E_\gamma)$ at certain energies (E_γ) obtained by eq. (1), a 2-D plot $\{\varepsilon(E_\gamma), E_\gamma\}$ is drawn. Scientists and technicians use various mathematical formulas to plot a curve through the calibrated full-energy peak efficiency values, which can later be used to calculate the gamma ray detector efficiency $\varepsilon(E_\gamma)$ at any other energy in the calibrated energy range when they analyze an unknown gamma spectrum.

MATHEMATICAL TREATMENT

Anyone working in the field of gamma spectroscopy should be aware that the experimental method is limited, this is due to many different reasons. One of the most important reasons may be the lack of calibrated radioactive sources due to their high cost and the decrease in their intensity due to their short half-life. Therefore, the detector efficiency for each peak in the measurement spectra, which is determined using calibrated radioactive sources, must be measured in the same geometry as soon as the detector reaches the laboratory to ensure that it covers the energy range of interest. After determining the efficiency $\varepsilon(E_\gamma)$,for each calibration peak, various mathematical functions depending on the gamma ray energy E can be proposed to fit the efficiency of the HPGe gamma detector. Many of these mathematical functions depend on the type of detector, the energy range covered by the source, and the distance from the source to the detector.

The fitting functions can be used only if their deviation from the experimentally obtained efficiency values for most energies and *source-to-detector* distances are very small. This can be done if the uncertainty of the measured full-energy peak efficiency, which is used in the weighted least squares (WLS) method, is very small. In this way, the fit parameters of the mathematical functions for all source-to-detector distances can be obtained with excellent precision. The weighting parameter is inversely proportional to the square of the efficiency uncertainty, $1/\sigma^2_{\varepsilon(E\gamma)}$. The energy range to be fitted must have closely spaced data points to obtain a continuous smooth fitting curve spanning the entire range, from which one can get the desired full-energy peak efficiency for any energies of interest.

Polynomial mathematical functions were regularly used on a logarithmic or ln scale, where their parameters were determined using the least-squares method after weighing the measured points. The degree of the polynomial, n, must be less than the number of measured data points, p. In some cases, two different fitting functions with a joint point can be used. In this case, a separate fit function is used for each energy range, and the result is two separate curves. Therefore, for a two-curve model, at least five calibration energy data points are required, two before the curves cross over point, one at the cross over point, and two after it. If one function is used, at least three measurement points are required to plot the curve and more points should be provided. In this study, to get the best results from plotting a smooth curve through experimental data in the recommended different range of gamma ray energies, several tested different mathematical functions such as

Mathematical functions cover the energy range from 59.54 up to 1408.01 keV (one region).

$$\ln\varepsilon(E_\gamma) = \sum_{i=0}^{n=4} a_i(\ln(E_\gamma))^i \quad \text{and}$$
$$\sigma^2_{\ln\varepsilon(E_\gamma)} = \left(\frac{1}{\varepsilon(E_\gamma)}\right)^2 \sigma^2_{\varepsilon(E_\gamma)} \tag{4}$$

$$\log\varepsilon(E_\gamma) = \sum_{i=1}^{n=4} a_i\left(\frac{b}{E_\gamma}\right)^i \quad \text{and}$$
$$\sigma^2_{\log\varepsilon(E_\gamma)} = \left(\frac{\log(e)}{\varepsilon(E_\gamma)}\right)^2 \sigma^2_{\varepsilon(E_\gamma)} \tag{5}$$

$$\ln\varepsilon(E_\gamma) = \sum_{i=0}^{n=4} a_i\left(\ln\frac{b}{E_\gamma}\right)^i \quad \text{and}$$
$$\sigma^2_{\ln\varepsilon(E_\gamma)} = \left(\frac{1}{\varepsilon(E_\gamma)}\right)^2 \sigma^2_{\varepsilon(E_\gamma)} \tag{6}$$

$$\log\varepsilon(E_\gamma) = \sum_{i=0}^{n=5} a_i[\log(E_\gamma)]^i \quad \text{and}$$
$$\sigma^2_{\log\varepsilon(E_\gamma)} = \left(\frac{\log(e)}{\varepsilon(E_\gamma)}\right)^2 \sigma^2_{\varepsilon(E_\gamma)} \tag{7}$$

$$\varepsilon(E_\gamma) = \sum_{i=0}^{n=7} a_i[\ln(E_\gamma)]^i / E_\gamma \tag{8}$$

and the variance is given as in eq. (3)

$$\varepsilon(E_\gamma) = \frac{1}{(aE_\gamma^b + cE_\gamma^d)} \tag{9}$$

and the variance is given as in eq. (3)

$$\log\frac{1}{\varepsilon(E_\gamma)} = \sum_{i=0}^{n=4} a_i\left(\frac{1}{E_\gamma}\right)^i \quad \text{and}$$
$$\sigma^2_{\log\varepsilon(E_\gamma)} = \left(\frac{\log(e)}{\varepsilon(E_\gamma)}\right)^2 \sigma^2_{\varepsilon(E_\gamma)} \tag{10}$$

Mathematical functions cover the energy range from 59.54 up to 1408.01 keV (two regions).

– First two regions:

$$\varepsilon(E_\gamma) = \sum_{i=0}^{n=2} a_i(E_\gamma)^i \tag{11}$$

range from 59.54 to 121.78 keV

$$\varepsilon(E_\gamma) = \sum_{i=0}^{n=2} a_i[\ln(E_\gamma)]^i + b/(E_\gamma)^3 \tag{12}$$

range from 121.78 to 1408.01 keV where the variance is given as in eq. (3)

– Second two regions:

$$\ln\varepsilon(E_\gamma) = \sum_{i=0}^{n=4} a_i(E_\gamma)^i \quad \text{and}$$
$$\sigma^2_{\ln\varepsilon(E_\gamma)} = \left(\frac{1}{\varepsilon(E_\gamma)}\right)^2 \sigma^2_{\varepsilon(E_\gamma)} \tag{13}$$

range from 59.54 to 223.24 keV

$$\ln\varepsilon(E_\gamma) = a\ln\left(\frac{c}{E_\gamma}\right) + b\left[\ln\left(\frac{c}{E_\gamma}\right)\right]^2 \quad \text{and}$$
$$\sigma^2_{\ln\varepsilon(E_\gamma)} = \left(\frac{1}{\varepsilon(E_\gamma)}\right)^2 \sigma^2_{\varepsilon(E_\gamma)} \tag{14}$$

range from 223.24 to 1408.01 keV

– Third two regions:

$$\varepsilon(E_\gamma) = \sum_{i=0}^{n=4} a_i(E_\gamma)^i \tag{15}$$

range from 59.54 to 276.4 keV

$$\varepsilon(E_\gamma) = \left(\frac{a}{(E_\gamma)}\right)^b \mathrm{e}^{E_\gamma} \tag{16}$$

range from 276.4 up to 1408.01 keV where the variance is given as in eq. (3).

In the above mathematical functions a_i, a, b, c, and d are the parameters of the fitted function, which are determined by the least-squares method, while the value $\sigma_{\ln(E)}$ is the variance of $\ln\varepsilon(E)$ and $\sigma_{\log\varepsilon(E)}$ is the variance of $\log\varepsilon(E)$. Based on the above equations, the weighting coefficient W_i for each measured point i is determined as

$$W_i = \frac{1}{Variance^2} \tag{17}$$

RESULTS AND DISCUSSIONS

The present study shows that all the mathematical functions used to describe the full-energy absorption peak efficiency curve of the HPGe gamma detector are consistent with each other to acceptable degrees. The full-energy absorption peak efficiency of the HPGe gamma ray detector was tested with thirteen mathematical functions, some with a cross-over point. Some problems are associated with the process of detecting gamma quanta with energies below 100 keV, where attenuation is one of them. The efficiency curve peaks at around 100 keV and then decreases exponentially towards higher energies. The full-energy absorption peak efficiency curves depend on the effective solid angle subtended by the detector with the point radioactive source, and on the intrinsic efficiency of the detector. At energies below 1.022 MeV, the intrinsic efficiency depends on the photoelectric and Compton cross-sections. Above energy of 1.022 MeV, the pair production cross-section also contributes to it.

When performing a nonlinear curve fit using the Origin laboratory program, an iterative procedure is employed to reduce the chi-square and obtain the optimal parameters values. The evaluation goodness of the fit of mathematical functions is based on the chi-square value χ^2, which provides a universal measure of the reliability of the measured efficiencies and their values from the fitted curve. In total 24 gamma rays energies from four radioactive point sources ^{60}Co, ^{133}Ba, ^{152}Eu, and ^{241}Am were used to determine the full-energy absorption peak efficiency of the HPGe gamma ray detector. Several different mathematical functions (formulas 4-16) were fitted to the experimentally obtained efficiency data points using the least-squares fit method. Seven different mathematical functions represented by eqs. (4)-(10) were fitted to the efficiency data points measured in the gamma ray energy range from 59.54 to 1408.01 keV. The measured efficiency data points along with their error bars are shown in fig. 2(a) up to fig. 2(g). Two methods were used to fit these measured points, the first using seven different mathematical functions to represent the full-energy peak efficiency of the HPGe gamma-ray detector over the entire energy range from 59.54 to 1408.01 keV, the least-squares fit parameters of these seven functions are shown in fig. 2.

The second method is to divide the entire energy region into two different regions using the cross-over (transition, intersection) point at energies of 121.78 keV, 223.24 keV, and 276.4 keV, respectively, as shown in fig. 3(a) up to fig. 3(c). The energy region before and after the cross-over point was fitted approximated by two different mathematical functions. The parameters related to these functions are shown in fig. 3. The residuals (discrepancies) between the measured points and the fitted curves are shown in figs. 2 and 3 based on the following equation

$$\text{Residual} \quad \frac{\varepsilon_{\text{exp}} \quad \varepsilon_{\text{Fit}}}{\varepsilon_{\text{exp}}} \, 100 \, [\%] \tag{18}$$

where ε_{exp} and ε_{fit} are the full-energy peak efficiency obtained experimentally and using the fitted functions, respectively.

The fluctuation of residuals is relatively small, their percentage is about 5 % in figs. 2 and 3, except for fig. 2(g) where it is less than 9 % when using eq. (10). The residual percentage is less than 6 % when using fit functions (11) and (12) as shown in fig. 3(a). The parameters related to these functions, obtained by the least-square method, are also indicated in the figures. In addition, tab. 1. Included the number of the equations that were used fitting, covered energy range, adjusted R^2, and reduced χ^2. When two intersected fit functions were used, the quality of the fit did not change significantly, these deviations were found to be acceptable for gamma-ray spectroscopy applications. In this study, it was shown that dividing the energy range and the piecewise approximation of the efficiency data leads to the same results. The results obtained are in good agreement with each other and show the applicability of the mathematical functions used to represent the full-energy absorption peak efficiency of the HPGe gamma-ray detector.

CONCLUSIONS

Thirteen mathematical functions were used to approximate the experimentally obtained full-energy absorption peak efficiency of the HPGe detector in the gamma ray energy range from 59.54 up to 1408.01 keV. The quality assessment of the mathematical fitting process was based on χ^2. The discrepancy between the measured points and the fitted curves was usually about 5 %. Based on the current investigation, one can say that these functions could be recommended for fitting the measured efficiency of the HPGe gamma ray spectrometer. They can also be used for testing different source-to-detector geometries and as permanent mathematical functions to approximate the efficiency of any gamma-ray detection system. This article summarizes the results of last year's work on improving useful methods for calibrating gamma-ray detectors in a wide energy range. The main goal of this study was to use a simple, economical, and correct procedure using the highest quality information collected.

AUTHOR'S CONTRIBUTIONS

M. S. Badawi and A. A. Thabet developed the mathematical model and B. A. Salem performed the numerical testing. All authors conceived and authored the article. M. S. Badawi and A. A. Thabet made valuable contributions in the experimental work and data analysis. All authors extensively interacted with each other, exchanging ideas, especially during the preparation of the manuscript.

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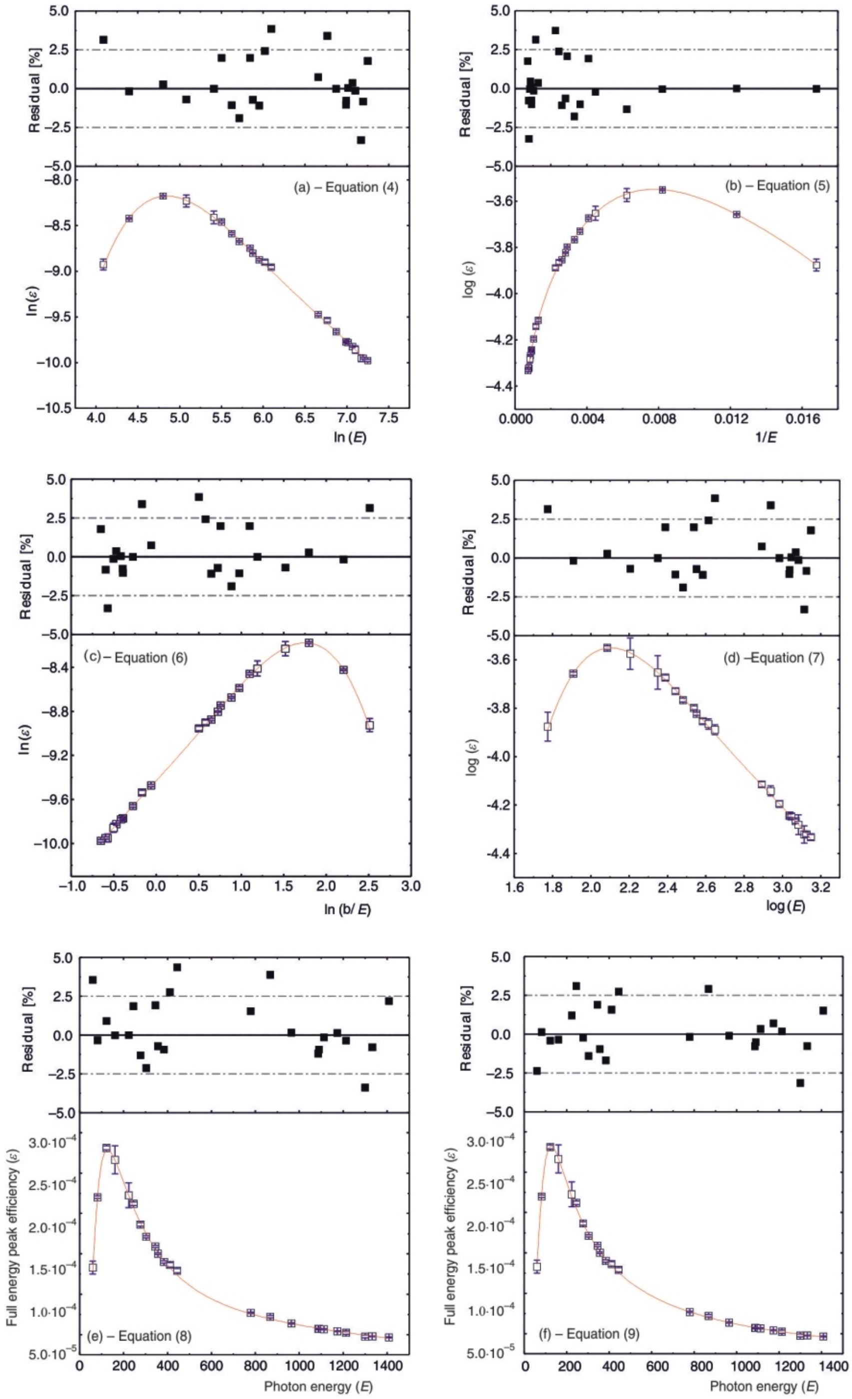

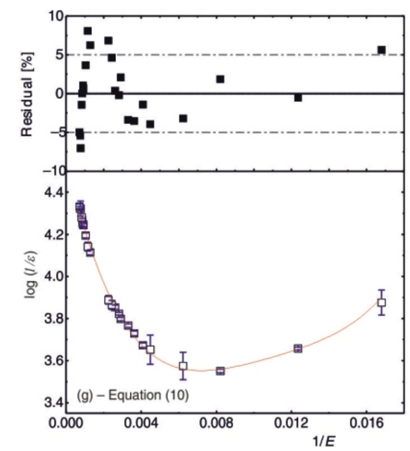

Figure 2. The measured efficiency data points in (a) up to (g) with their error bars and its fitting values that represent one energy region by using eqs. (4)-(10), beside the residual percentage

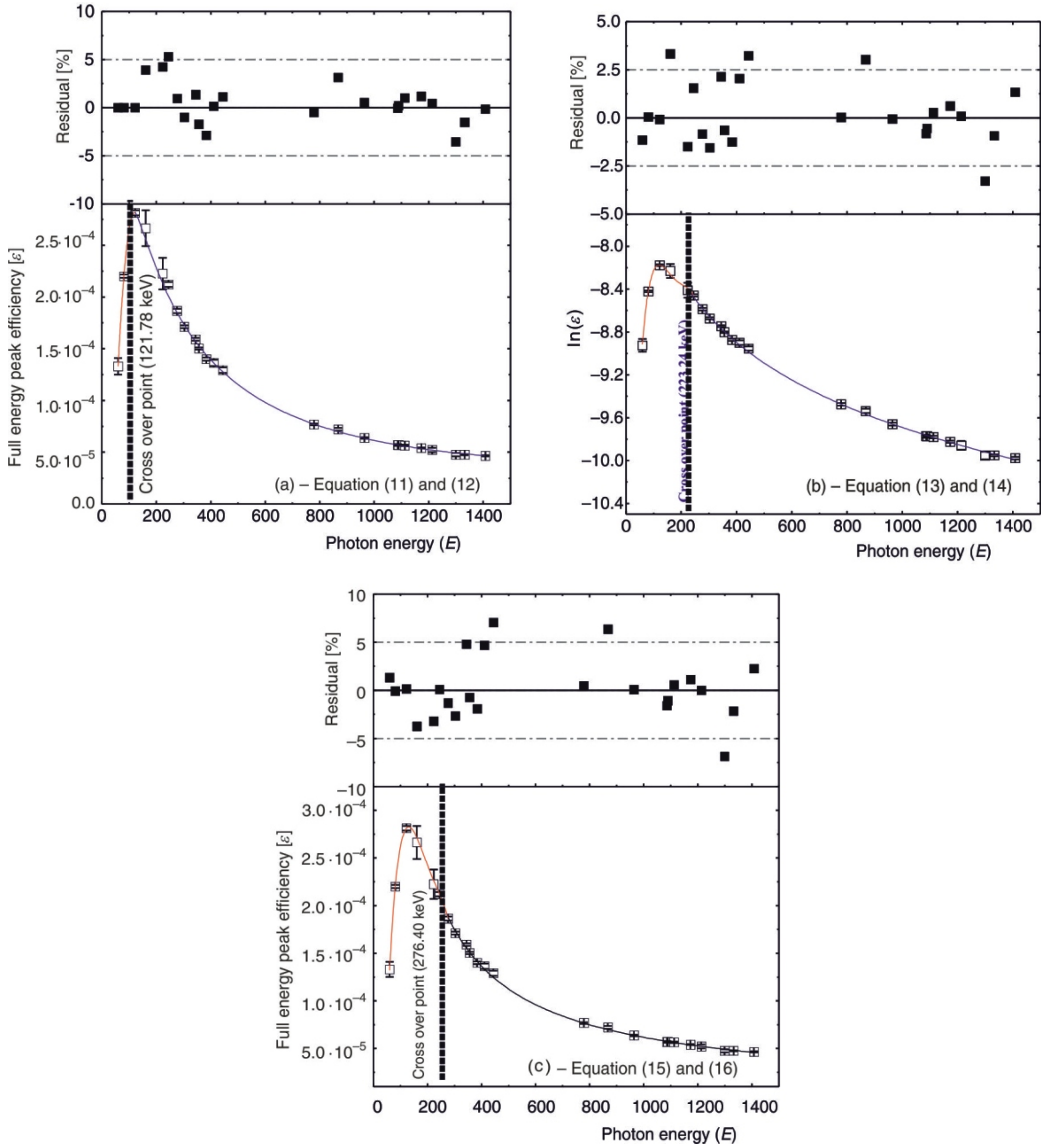

Figure 3. The measured efficiency data points in (a) up to (c) with their error bars and its fitting values that represent one energy region by using eqs. (11)-(16), beside the residual percentage

Table 1. The number of the equations that were used fitting, covered energy range, adjusted R^2, and reduced Chi-square

Equation number	Reduced χ^2	Adjust R^2	Covered energy range [keV]	
			From	To
4	1.64855	0.99947		
5	1.61472	0.99948		
6	1.64855	0.99947		
7	0.32055	0.99945	59.54	1408.01
8	2.12257	0.99902		
9	1.49009	0.99931		
10	1.64686	0.99718		
11	3.71E-39	1	59.54	121.78
12	2.90E+00	0.99849	121.78	1408.01
13	0.40326	0.99714	59.54	223.24
14	1.45571	0.99941	223.24	1408.01
15	0.08095	0.99953	59.54	276.40
16	1.40996	0.99915	276.40	1408.01

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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ПРОУЧАВАЊЕ МАТЕМАТИЧКИХ ФУНКЦИЈА ЗА ФИТОВАЊЕ КРИВИХ ЕФИКАСНОСТИ ДЕТЕКТОРА У ОБЛАСТИ ЕНЕРГИЈА ГАМА ЗРАЧЕЊА ОД 59.54 keV ДО 1408.01 keV

У новије време било је неколико значајних побољшања система и инструментације детекције зрачења, посебно оних које користе сцинтилационе или полупроводничке детекторе гама зрачења. Истраживачи и техничари заинтересовани су за проучавање овог развоја, што је од користи за управљање детекторима и њиховим основним својствима, као што су енергија, облик и калибрација ефикасности. У овом раду спроведено је обухватно проучавање различитих математичких формула ради добијања најбољих апроксимација функција ефикасности, које покривају мерене вредности у областима од ниских до високих енергија. Оне могу бити коришћене да опишу ефикасност германијумског детектора високе чистоће у областима у којима су тачност и максимална брзина оптимизације калибрационог процеса веома значајне за гама спектроскопију. Одређивање активности узорака из околине углавном зависи од ефикасности калибрационих кривих детекторског система. Енергија гама зрачења од 59.54 keV до 1408.01 keV коришћена у овом раду добијена је сетом стандардних радиоактивних извора гама зрачења сертификованог интензитета. Анализа података показала је да је већина математичких формула, које апроксимирају криве ефикасности детектора у пику укупне енергије, сасвим сагласна са експерименталним резултатима.

Кључне речи: математичка формула, ефикасност у пику укупне енергије, процес фитовања, детектор гама зрачења, радиоактивни тачкасти извор