LINEAR STABILITY ANALYSIS OF A NUCLEAR REACTOR USING THE LUMPED MODEL

by

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The stability analysis of a nuclear reactor is an important aspect in the design and operation of the reactor. A stable neutronic response to perturbations is essential from the safety point of view. In this paper, a general methodology has been developed for the linear stability analysis of nuclear reactors using the lumped reactor model. The reactor kinetics has been modelled using the point kinetics equations and the reactivity feedbacks from fuel, coolant and xenon have been modelled through the appropriate time dependent equations. These governing equations are linearized considering small perturbations in the reactor state around a steady operating point. The characteristic equation of the system is used to establish the stability zone of the reactor considering the reactivity coefficients as parameters. This methodology has been used to identify the stability region of a typical pressurized heavy water reactor. It is shown that the positive reactivity feedback from xenon narrows down the stability region. Further, it is observed that the neutron kinetics parameters (such as the number of delayed neutron precursor groups considered, the neutron generation time, the delayed neutron fractions, *etc*.) do not have a significant influence on the location of the stability boundary. The stability boundary is largely influenced by the parameters governing the evolution of the fuel and coolant temperature and xenon concentration.

Key words: linear stability, PHWR, reactivity coefficient, xenon feedback, stability region

INTRODUCTION

The stable neutronic response of a nuclear reactor is one of the important requirements for the safe reactor operation. During the course of its operation, the reactor is subjected to many intentional or unintentional perturbations which tend to change the rector power from its steady operating value. These perturbations include the movement of the reactivity control devices, changes in the coolant poison concentration, fluctuations in the coolant flow, variations in the power demand etc. In response to such perturbations, a stable reactor returns back to the steady-state. If the reactor is not stable, it moves away from the steady-state following a perturbation and this may lead to unwanted deviations in the safety parameters or even lead to accident conditions if not controlled. Therefore, thorough analysis of reactor stability is necessary to demonstrate its ability to withstand the reactivity perturbations.

Point reactor kinetics has been conventionally used for the stability analysis of reactors [1, 2]. In the present work, a general methodology has been developed for the linear stability analysis of nuclear reactors, where in addition to the fuel and coolant temperature reactivity feedbacks, the xenon reactivity feedback has also been included. A methodology has been established for identifying the stability region of a nuclear reactor, *i. e*., for identifying the region of permissible values of reactivity coefficients for stable operation. The equations governing the evolution of reactor power, fuel and coolant temperatures and xenon reactivity are based on the point reactor model. These equations are linearized considering small deviations around the steady operating state. The characteristic equation of the system is obtained and used for identifying the stability region. This methodology has been illustrated for a typical pressurized heavy water reactor (PHWR). The stability boundary has been identified for this reactor. The influence of point kinetics parameters and feedback mechanisms on stability of the reactor has been studied. The role of xenon reactivity feedback on the stability of PHWR is brought out.

MODELLING

The lumped model representing the transient evolution of the reactor has been used in the present study for analysing the stability of the reactor core *i. e*., the

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equations governing the neutron kinetics, reactor thermal hydraulics and xenon dynamics are independent of space. This approximation simplifies the calculations while incorporating all the necessary physical aspects of reactor dynamics. Although it does not address the spatial effects, it is useful for the quick assessment of global stability aspects of nuclear reactors.

Reactor model

The neutron kinetics is modelled through point kinetics equations with six groups of delayed neutron precursors

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{\rho - \beta}{\Lambda} P + \sum_{i=1}^{6} \lambda_i C_i \tag{1}$$

$$\frac{\mathrm{d}C_i}{\mathrm{d}t} = \frac{\beta_i}{\Lambda} p - \lambda_i C_i \quad i = 1,2,\ldots,6 \tag{2}$$

The reactivity ρ, among other things, depends on the cross-sections, reactor geometry and flux distribution [3]. In general, it is a complex function of reactor power. Its presence in eq. (1) introduces a non-linearity and the equations need to be solved numerically. However, the equations can be linearized by considering small perturbations in the variables around their equilibrium values so that linear stability analysis techniques can be employed. This approach has been discussed in the following sections. Although this method cannot be applied to arbitrary reactivity perturbations, it can be used for the assessment of reactor's stability in the vicinity of the steady operating point.

The instantaneous reactivity $\rho(t)$ can be expressed as the sum of external reactivity $\rho_{\mathrm{ext}}(t)$ (*e. g.* due to the movement of control rods, changes in coolant poison concentration, *etc*.) and the reactivity feedbacks from the system (*e. g.* due to the changes in fuel and coolant temperature, reactor poison concentration, *etc*.)

$$\rho(t) = \rho_{\mathrm{ext}}(t) + \rho_{\mathrm{fb}}(t) \tag{3}$$

For the steady-state (critical) operation of a reactor

$$\rho(t = 0) = \rho_{\mathrm{ext}}(0) + \rho_{\mathrm{fb}}(0) = 0 \tag{4}$$

For small deviations around the steady-state,

$$\delta\rho_{\mathrm{ext}}(t) = \rho_{\mathrm{ext}}(t) - \rho_{\mathrm{ext}}(0) \tag{5}$$

and

$$\delta\rho_{\mathrm{fb}}(t) = \rho_{\mathrm{fb}}(t) - \rho_{\mathrm{fb}}(0) \tag{6}$$

thus

$$\delta\rho(t) = \rho(t) - \rho(0) = \rho(t) - \delta\rho_{\mathrm{fb}} - \delta\rho_{\mathrm{ext}} \tag{7}$$

For a given steady operating power P_0, the steady-state precursor concentrations C_{i0}* can be obtained by equating the RHS of eq. (2) to zero. If a small change in reactivity is introduced, the reactor state tends to deviate from the steady-state. A stable reactor will eventually return to the steady-state while an unstable reactor moves away from the steady-state. For small deviations around the steady-state, the following non-dimensional variables can be defined

$$x = \frac{P - P_0}{P_0} \quad P = P_0(x + 1) \tag{8}$$

$$y_i = \frac{C_i - C_{i0}}{C_{i0}} \quad C_i = C_{i0}(y_i + 1) \tag{9}$$

Substituting (8) and (9) in eq. (1) and using the steady-state condition, the following equation is obtained

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\rho x}{\Lambda} + \frac{\beta}{\Lambda} x + \frac{1}{\Lambda} \sum_{i=1}^{6} \beta_i y_i + \frac{\rho}{\Lambda} \tag{10}$$

For small values of reactivity ρ, since ρx is a product of two small quantities, it can be neglected. With this assumption, eq. (10) becomes

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\beta}{\Lambda} x + \frac{1}{\Lambda} \sum_{i=1}^{6} \beta_i y_i + \frac{\rho}{\Lambda} \tag{11}$$

Similarly, substituting eqs. (8) and (9) in eq. (2) and using the steady-state condition

$$\frac{\mathrm{d}y_i}{\mathrm{d}t} = \lambda_i x - \lambda_i y_i \quad i = 1,2,\ldots,6 \tag{12}$$

Thus, for small reactivity perturbations around the steady-state, the reactor behaviour can be considered linear and the linear stability analysis methods can be employed. Taking Laplace transforms* of eqs. (11) and (12) and simplifying, the reactor transfer function is obtained

$$G(s) = \frac{x(s)}{\rho(s)} = \frac{1}{s\Lambda - s\sum_{i=1}^{6} \frac{\beta_i}{s + \lambda_i}} \tag{13}$$

The calculations can be simplified by defining the equivalent one group of delayed neutron precursors with the decay constant defined as [4]

$$\lambda = \frac{\beta}{\sum_{i=1}^{6} \frac{\beta_i}{\lambda_i}} \tag{14}$$

where

$$\beta = \sum_{i=1}^{6} \beta_i \tag{15}$$

For one group of delayed neutron precursors, the point kinetics equations become

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{\rho - \beta}{\Lambda} P + \lambda C \tag{16}$$

* In the equations written here C_i is actually the quantity proportional to the precursor concentration since the point kinetics equations are written in terms of power and not in terms of neutron density

* For convenience, the Laplace transforms of functions are denoted by the same functional notation, *e. g.* $L[x(t)] = x(s), L[z_{\mathrm{f}}(t)] = z_{\mathrm{f}}(s)$, *etc.*

$$\frac{dC}{dt} = \frac{\beta}{\Lambda}P - \lambda C \tag{17}$$

Using the non-dimensional power and precursor concentrations from eqs. (8) and (9) and Laplace transforming eqs. (16) and (17), the reactor transfer function for one group of delayed neutron precursors is obtained

$$G(s) = \frac{x(s)}{\rho(s)} = \frac{s + \lambda}{s\Lambda + s + \lambda - \frac{\beta}{\Lambda}} \tag{18}$$

Reactivity feedbacks

Changes in reactor power due to external perturbations lead to changes in the fuel and the coolant state and the reactor poison concentration. These changes (*e. g.* variations in fuel and coolant temperature, coolant density, local changes in fuel-coolant configuration *etc*.) introduce reactivity changes or reactivity feedbacks. Important reactivity feedback mechanisms for pressurised water/heavy water reactors (PWR/PHWR) includes fuel and coolant temperature effects and xenon reactivity feedbacks. The reactivity changes associated with the fuel and coolant temperature variation can be calculated through corresponding reactivity coefficients and temporal evolution of fuel and coolant temperature. The xenon reactivity effects can be calculated using the equations governing the variation of ^{135}Xe concentration in time.

A small change in the feedback reactivity due to the changes in fuel and coolant temperatures and xenon concentration can be expressed as [3]

$$\delta\rho_{fb} = \alpha_f \delta T_f + \alpha_c \delta T_c + \delta\rho_X \tag{19}$$

The reactivity coefficients α_f and α_c are defined by

$$\alpha_f = \frac{\partial\rho}{\partial_f} \qquad \alpha_c = \frac{\partial\rho}{\partial_c} \tag{20}$$

Fuel and coolant dynamics

The equations governing the temporal variation of fuel and the coolant temperature in a reactor coolant channel can be obtained from the energy balance between the fuel pin and the coolant flowing outside

$$\frac{dT_f}{dt} = a_1 P + a_2 (T_f - T_c) \tag{21}$$

$$\frac{dT_c}{dt} = a_3 (T_f - T_c) + a_4 (T_c - T_{c_{in}}) \tag{22}$$

where T_f and T_c are the average fuel and coolant temperatures in the core respectively. $T_{c_{in}}$ is the coolant temperature at the inlet of the reactor core. a_1, a_2, a_3, and a_4 are constants at the given steady operating state and are defined by

$$a_1 = \frac{1}{m_f C_f}, a_2 = \frac{h_{eq} A_{eq}}{m_f C_f}, a_3 = \frac{h_{eq} A_{eq}}{m_c C_c}, a_4 = \frac{2\dot{m}_{flow}}{m_c} \tag{23}$$

where h_{eq} is the equivalent heat transfer coefficient between the fuel and the coolant corresponding to the area A_{eq}. The effect of thermal resistance due to the pellet-clad gap, the clad thickness and the coolant film outside the fuel pin has been accounted through the equivalent heat transfer coefficient h_{eq}. The steady-state values of fuel and coolant temperature (T_{f0}, T_{c0}) are obtained by equating the right hand side of eqs. (21) and (22) to zero.

For small deviations around the steady-state, the following non-dimensional temperatures are defined

$$z_f = \frac{T_f - T_{f0}}{T_{f0}} \qquad T_f = T_{f0}(z_f + 1) \tag{24}$$

$$z_c = \frac{T_c - T_{c0}}{T_{c0}} \qquad T_c = T_{c0}(z_c + 1) \tag{25}$$

Substituting eqs. (24) and (25) in eqs. (21) and (22), the following governing equations in normalized form are obtained

$$T_{f0}\frac{dz_f}{dt} = a_1 P_0 x + a_2 (T_{f0} z_f - T_{c0} z_c) \tag{26}$$

$$T_{c0}\frac{dz_c}{dt} = a_3 T_{f0} z_f - (a_3 + a_4) T_{c0} z_c \tag{27}$$

Taking the Laplace transforms of eqs. (26) and (27) and using $z_f(0) = 0$ and $z_c(0) = 0$, the transfer functions for fuel and coolant reactivity feedbacks are obtained

$$H_f(s) = \frac{z_f(s)}{x(s)} = \frac{a_1 (a_3 + a_4 + s)}{s^2 + s(a_2 + a_3 + a_4) + a_2 a_4} \frac{P_0}{T_{f0}} \tag{28}$$

$$H_c(s) = \frac{z_c(s)}{x(s)} = \frac{a_1 a_3}{s^2 + s(a_2 + a_3 + a_4) + a_2 a_4} \frac{P_0}{T_{c0}} \tag{29}$$

Xenon feedback

Xenon-135 (^{135}Xe) is a product of fission which has very high capture affinity for thermal neutrons. It acts as a neutron poison due to its large neutron absorption cross section and it can have a significant influence on slow power transients in thermal reactors as it introduces reactivity changes due to changes in its concentration in the core. The modes of production and removal of ^{135}Xe are schematically shown in fig. 1.

The half life of Tellurium is very small (~19 s); therefore it can be assumed that iodine is produced di-

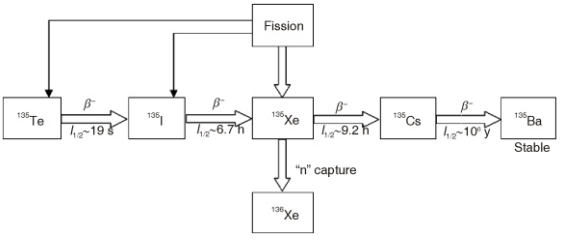

Figure 1. Modes of production and removal of ^{135}Xe

rectly from fission (fission yield γ_I in eq. (34) accounts for both ^{135}Te and ^{135}I). The equations governing the temporal variation of reactivity due to the poison can be written

$$\frac{d\rho_I}{dt} \quad a_5 P \quad \lambda_I \rho_I \tag{30}$$

$$\frac{d\rho_X}{dt} \quad a_6 P \quad \lambda_I \rho_I \quad \lambda_X \rho_X \quad a_7 \rho_X P \tag{31}$$

where the reactivity equivalent of xenon (ρ_x) is defined [4]

$$\rho_X \quad \frac{\sigma_X C_X}{F\Sigma_a} \tag{32}$$

and ρ_I is a quantity proportional to the iodine concentration defined

$$\rho_I \quad \frac{\sigma_X C_I}{F\Sigma_a} \tag{33}$$

where σ_X is the microscopic neutron absorption cross-section of xenon (^{135}Xe), C_X and C_I are xenon and iodine concentrations, Σ_a is the total macroscopic neutron absorption cross-section, F is the correction factor to account for the finite size of the reactor and fast neutron production, and a_5, a_6, and a_7 are constants defined

$$a_5 \quad \frac{\gamma_I \quad {}^f_t\sigma_X}{{}_a F}\frac{\phi_{ref}}{P_{ref}}, \quad a_6 \quad \frac{\gamma_X \quad {}^f_t\sigma_X}{{}_a F}\frac{\phi_{ref}}{P_{ref}}, \quad a_7 \quad \frac{\sigma_X \phi_{ref}}{P_{ref}} \tag{34}$$

where ϕ_{ref} is the reference effective thermal neutron flux at power P_{ref}.

For small deviations around the steady-state (ρ_{I0}, ρ_{X0}), normalized quantities z_I and z_X are defined as

$$z_I \quad \frac{\rho_I \quad \rho_{I0}}{\rho_{I0}} \quad \rho_I \quad \rho_{I0}(z_I \quad 1) \tag{35}$$

$$z_X \quad \frac{\rho_X \quad \rho_{X0}}{\rho_{X0}} \quad \rho_X \quad \rho_{X0}(z_X \quad 1) \tag{36}$$

Substituting eqs. (35) and (36) in eqs. (30) and (31), the following equations are obtained

$$\frac{dz_I}{dt} \quad \lambda_I x \quad \lambda_I z_I \tag{37}$$

$$\begin{aligned} \frac{dz_X}{dt} \quad & a_6 x \frac{P_0}{\rho_{X0}} \quad \lambda_I z_I \frac{\rho_{I0}}{P_{X0}} \\ & \lambda_X z_X \quad a_7 P_0(z_X x \quad z_X \quad x) \\ \frac{dz_X}{dt} \quad & a_6 x \frac{P_0}{\rho_{X0}} \quad \lambda_I z_I \frac{\rho_{I0}}{\rho_{X0}} \\ & \lambda_X z_X \quad a_7 P_0(z_X \quad x) \end{aligned} \tag{38}$$

where $z_X x$ is neglected as it is a product of two small quantities. By taking the Laplace transforms of eqs. (37) and (38) and simplifying, the transfer function for xenon reactivity feedback is obtained

$$H_X(s) \quad \frac{z_X(s)}{x(s)} \quad \frac{[a_5\lambda_I \quad (a_6 \quad a_7\rho_{X0})(s \quad \lambda_I)]}{(s \quad \lambda_I)(s \quad \lambda_X \quad a_7 P_0)} \frac{P_0}{\rho_{X0}} \tag{39}$$

where ρ_{X0} is the steady-state xenon reactivity load at power P_0.

Overall transfer function

Substituting eqs. (24), (25), and (36) in eq. (19), a change in feedback reactivity can be obtained

$$\delta\rho_{fb} \quad \alpha_f T_{f0} z_f \quad \alpha_c T_{c0} z_c \quad \rho_{X0} z_X \tag{40}$$

The Laplace transformation of eq. (40) results in

$$\delta\rho_{fb}(s) \quad \alpha_f T_{f0} z_f(s) \quad \alpha_c T_{c0} z_c(s) \quad \rho_{X0} z_X(s) \tag{41}$$

Using eqs. (28), (29), and (39)

$$\begin{aligned} \delta\rho_{fb}(s) \quad & \alpha_f T_{f0} H_f(s) x(s) \\ & \alpha_c T_{c0} H_c(s) x(s) \quad \rho_{X0} H_X(s) x(s) \end{aligned} \tag{42}$$

The overall reactivity feedback transfer function $H(s)$ is

$$\begin{aligned} H(s) \quad & \frac{\delta\rho_{fb}(s)}{x(s)} \quad \alpha_f T_{f0} H_f(s) \\ & \alpha_c T_{c0} H_c(s) \quad \rho_{X0} H_X(s) \end{aligned} \tag{43}$$

Also, the Laplace transformation of eq. (7) gives

$$\rho(s) \quad \delta\rho_{ext}(s) \quad \delta\rho_{fb}(s) \tag{44}$$

The closed loop block diagram for the overall system with different feedback mechanisms is shown in fig. 2. The equivalent block diagram is shown in fig. 3.

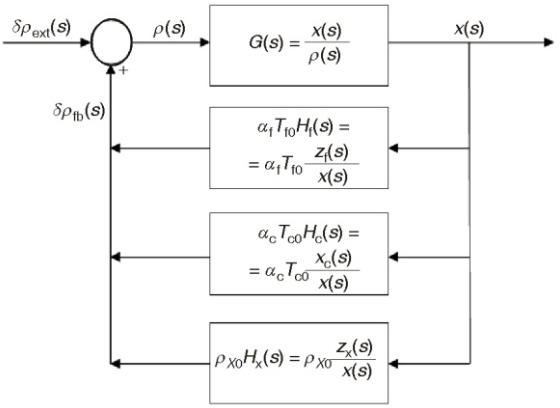

Figure 2. Closed loop block diagram with different feedback mechanisms

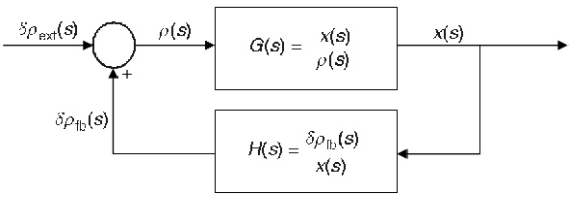

Figure 3. Overall closed loop block diagram

The overall closed loop transfer function for the system is given as

$$Tr(s) \quad \frac{G(s)}{1 \quad G(s)H(s)} \tag{45}$$

DETERMINING THE STABILITY OF THE REACTOR

The Laplace transform approach

The condition for stability of the closed loop system is that the roots of the characteristic equation

$$\begin{array}{cccccc} 1 & G(s)H(s) & b_{\mathrm{n}}s^{n} & b_{\mathrm{n}\ 1}s^{n\ 1} & \cdots \\ & & b_2 s^2 & b_1 s & b_0 & 0 \end{array} \tag{46}$$

must all have negative real parts *i. e.* the poles of the overall transfer function $Tr(s)$ must lie in the left (negative) half of the complex s plane [5]. Thus, for given values of reactivity coefficients α_{f} and α_{c}, the stability of the reactor system can be assessed by directly calculating the poles of transfer function or by using linear stability analysis techniques such as Nyquist plot, Bode plots, Routh-Hurwitz criterion, *etc*.

This method of determining the stability of the system for given reactivity coefficients can be further extended to identify the stability region of the reactor with respect to the reactivity coefficients α_{f} and α_{c}, *i. e.*, for a given reactor, the stability boundary can be established by considering α_{f} and α_{c} as parameters. This can be achieved using the Routh-Hurwitz criterion [5]. The Routh-Hurwitz method can be used to identify the nature of roots without actually calculating the roots of the characteristic equation.

For no root of the characteristic eq. (46) to have positive real part, the necessary condition is that the coefficients b_{n}, $b_{\mathrm{n}-1}$, ..., b_1, b_0 have the same sign and no coefficient vanishes. In general, the necessary and sufficient condition (Routh-Hurwitz criterion) for all the roots of the characteristic equation to lie in the left half of s-plane is that there is no sign change in the first column of the routh array. Since α_{f} and α_{c} are taken as parameters, coefficients $\mathrm{b_n}$, $b_{\mathrm{n}-1}$, ..., b_1, b_0 and the entries in the first column of routh array are functions of α_{f} and α_{c}. By requiring that there is no sign change in the coefficients $\mathrm{b_n}$, $b_{\mathrm{n}-1}$, ..., b_1, b_0 and in the first column of the routh array (*i. e.* all the entries are either positive or negative), an inequality is obtained for each entry. The intersection of plots of these inequalities in the α_{f} *vs.* α_{c} plane gives the stability region of the reactor.

Eigenvalue approach

An equivalent method for determining the stability of the system is through the eigenvalues* of the system. Equations (11), (12), (26), (27), (37), and (38) can be written in the matrix from as

$$\frac{\mathrm{d}Y(t)}{\mathrm{d}t} \quad \underline{A}Y(t) \quad R(t) \tag{47}$$

where Y is the reactor state vector, $\underline{A}$ is the matrix of constants and $R(t)$ is the input vector that depends on the value of external reactivity $\delta\rho_{\mathrm{ext}}$. Equations (7) and (40) have been used for calculation of total reactivity $\rho\ t$

$$Y \quad [x\ y_1 \ldots y_6\ z_{\mathrm{f}}\ z_{\mathrm{c}}\ z_{\mathrm{I}}\ z_{\mathrm{X}}] \tag{48}$$

$$\underline{A} \quad \begin{array}{ccccccccc} \frac{\beta}{\Lambda} & \frac{\beta_1}{\Lambda} & . & . & . & \frac{\beta_6}{\Lambda} & \frac{\alpha_{\mathrm{f}}T_{\mathrm{f0}}}{\Lambda} & \frac{\alpha_{\mathrm{c}}T_{\mathrm{c0}}}{\Lambda} & 0 & \frac{\rho_{\mathrm{X0}}}{\Lambda} \\ \lambda_1 & \lambda_1 & . & . & . & 0 & 0 & 0 & 0 & 0 \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . \\ \lambda_6 & 0 & . & . & . & \lambda_6 & 0 & 0 & 0 & 0 \\ \frac{a_1 P_0}{T_{\mathrm{f0}}} & 0 & . & . & . & 0 & a_2 & \frac{a_2 T_{\mathrm{c0}}}{T_{\mathrm{f0}}} & 0 & 0 \\ 0 & 0 & . & . & . & 0 & \frac{a_3 T_{\mathrm{f0}}}{T_{\mathrm{c0}}} & -(a_3 \quad a_4) & 0 & 0 \\ \lambda_{\mathrm{I}} & 0 & . & . & . & 0 & 0 & 0 & \lambda_{\mathrm{I}} & 0 \\ \frac{a_6 P_0}{\rho_{\mathrm{X0}}} & a_7 P_0 & 0 & . & . & . & 0 & 0 & \frac{\lambda_{\mathrm{I}}\rho_{\mathrm{I0}}}{\rho_{\mathrm{X0}}} & (\lambda_X \quad a_7 P_0) \end{array} \tag{49}$$

$$R(t) \quad \frac{\delta\rho_{\mathrm{ext}}(t)}{\Lambda} 0 \quad \ldots \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \tag{50}$$

The condition for the system to be stable is that the eigenvalues of the coefficient matrix $\underline{A}$ have negative real parts [5]. Again, taking α_{f} and α_{c} as parameters (all other matrix elements are constants),

* Eigenvalues are the roots of the characteristic equation $|\underline{A} \quad s\underline{I}| \quad 0$, where $\underline{I}$ is the identity matrix.

eigenvalues of are calculated. These eigenvalues are functions of α_f and α_c whose real parts should be less than zero for stability. Plots of the inequalities thus obtained give the stability region in the $_f$-α_c plane.

Apart from the Laplace transform and eigenvalue approach, the stability region can also be found directly by sweeping the entire range of interest of values of α_f and α_c and identifying the threshold points (α_f, α_c) where the reactor becomes unstable (the threshold points are the values α_f and α_c where real parts of the poles/eigenvalues become positive).

STABILITY ANALYSIS OF A TYPICAL PHWR

The methodology discussed here can be used for linear stability analysis of thermal reactors which use single phase coolant. This is illustrated here for a typical PHWR [6]. The PHWR is a horizontal pressure tube type reactor which uses natural uranium fuel. D_2O is used as coolant as well as the moderator which are physically separated. The point kinetics parameters [7] for the reactor are listed in tab. 1.

Thermal hydraulic constants, in eq. (21) and (22), for lumped fuel and coolant model [6, 8] are given in tab. 2. The parameters used in xenon reactivity calculations [3, 4] are presented in tab. 3.

The steady-state fuel and coolant temperatures and xenon reactivity load for full power operation at 756 MW_t are given in tab. 4.

The values listed in tab. 1 to tab. 4 correspond to the full power operation for equilibrium core of the reactor and are assumed constant for the present analysis. Using these values, the overall transfer function and characteristic equation of the system is obtained and the stability region for the reactor is plotted using the methodology discussed in the above sections. In order to assess the influence of xenon reactivity feedback on reactor stability, the stability maps have been obtained for two different cases – (1) without the xenon reactivity feedback and (2) with the xenon reactivity feedback.

Table 1. Point kinetics parameters for PHWR

Delayed neutron fractions (β_i	1.89, 11.3, 10.3, 21.1, 7.56, 1.89
Decay constants (λ_i) [s^{-1}]	0.0128, 0.0315, 0.122, 0.315, 1.386, 3.466
One group delayed neutron fraction (β)	0.0054
One group decay constant (λ) [s^{-1}]	0.0812
Neutron generation time (Λ) [s]	$7.0\ 10^{-4}$

Table 2. Thermal-hydraulic constants

a_1 [°CJ^{-1}]	$6.67 \cdot 10^{-8}$
a_2 [s^{-1}]	0.142
a_3 [s^{-1}]	0.169
a_4 [s^{-1}]	2.865
Coolant inlet temperature (T_{cin}), [°C]	249

Table 3. Xenon dynamics constants

Iodine decay constant (λ) [s^{-1}]	$2.94\ 10^{-5}$
Xenon decay constant (λ_X) [s^{-1}]	$2.1\ 10^{-5}$
a_5 [J^{-1}]	$8.0\ 10^{-15}$
a_6 [J^{-1}]	$7.77\ 10^{-16}$
a_7 [J^{-1}]	$2.91\ 10^{-13}$

Table 4. Steady-state values

Thermal power (P_0) [W]	$756\ 10^6$
Average fuel temperature (T_{f0}) [°C]	626
Average coolant temperature (T_{c0}) [°C]	270
Xenon reactivity (ρ_{x0}) [mk]*	–27.7

* mk stands for 10^{-3}

RESULTS

Stability maps (one group of precursors)

The characteristic equation for the case without the xenon feedback is

$$\begin{aligned} &4.46\ 10^{\ 6}s^4 \quad 4.89\ 10^{\ 5}s^3 \\ &(0.321\alpha_f \quad 1.12\ 10^{\ 4})s^2 \\ &(\alpha_f \quad 5.42\ 10^{\ 2}\alpha_c \quad 1.41\ 10^{\ 5})s \\ &(7.91\ 10^{\ 2}\alpha_f \quad 4.40\ 10^{\ 3}\alpha_c) \quad 0 \end{aligned} \tag{51}$$

Since the first two coefficients are negative, we require that the next three coefficients are also negative so that the roots have negative real parts. Further, after constructing the routh array, we also require that there should be no sign change in the first column of the routh array for the real parts of the roots of this equation (*i. e.* eigenvalues of the coefficient matrix or poles of the system transfer function) to be negative. The plots of inequalities thus obtained give the stability map. The stability map for this case (where only fuel and coolant temperature feedbacks are considered – no xenon feedback) is shown in fig. 4. The shaded portion indicates the region of stability.

The characteristic equation for the case where the xenon feedback is also included becomes

$$\begin{aligned} &4.46\ 10^{\ 6}s^6 \quad 4.89\ 10^{\ 5}s^5 \\ &(0.321\alpha_f \quad 1.12\ 10^{\ 4})s^4 \\ &(\alpha_f \quad 5.42\ 10^{\ 2}\alpha_c \quad 1.41\ 10^{\ 5})s^3 \\ &(7.94\ 10^{\ 2}\alpha_f \quad 4.42\ 10^{\ 3}\alpha_c \quad 1.93\ 10^{\ 8})s^2 \\ &(2.14\ 10^{\ 5}\alpha_f \quad 1.19\ 10^{\ 6}\alpha_c \quad 1.15\ 10^{\ 9})s \\ &(5.59\ 10^{\ 10}\alpha_f \quad 3.11\ 10^{\ 11}\alpha_c \quad 3.57\ 10^{\ 15}) \quad 0 \end{aligned} \tag{52}$$

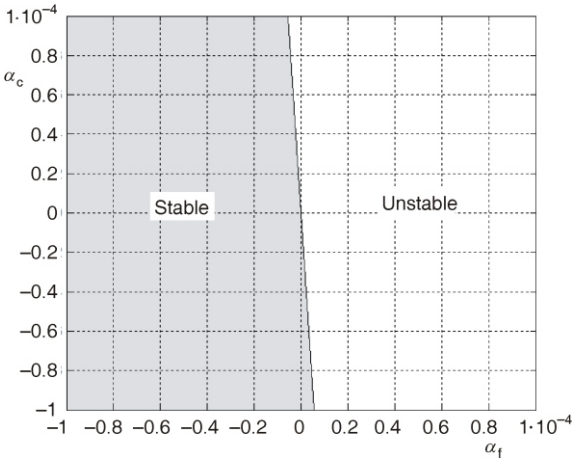

Figure 4. Stability map when xenon feedback effects are neglected

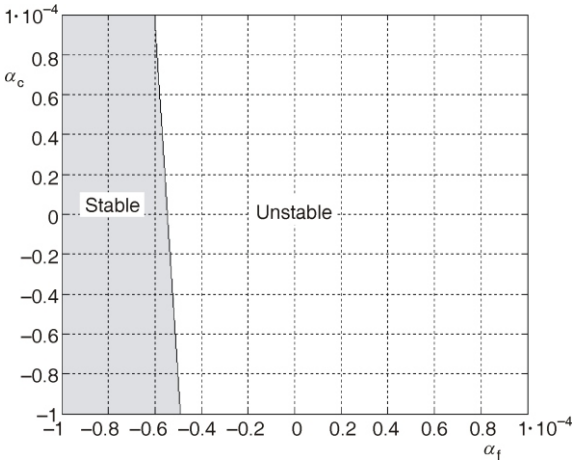

Figure 5. Stability map with xenon feedback effects included

The stability map for this case is shown in fig. 5.

It is clear that when xenon reactivity effect is neglected, the reactor is stable for negative values of reactivity coefficients. The slope of the stability boundary indicates that the stability is largely influenced by the changes in the fuel temperature coefficient and to a less extent by the changes in coolant temperature coefficient. This is consistent with the fact that the fuel temperature reactivity effect is a prompt effect whereas the coolant reactivity feedback is a delayed effect. The inclusion of xenon reactivity feedback shifts the stability boundary to the left in α_f *vs.* α_c plane, thus narrowing down the stability region.

Stability maps (six groups of precursors)

The inclusion of six groups of delayed neutron precursors in the point kinetics equations increases the degree of characteristic polynomial and subsequent complexity of stability calculations. However, in order to assess the accuracy of one group approximation, the calculations were also carried out with six group precursors. The stability boundaries obtained using with six group precursor equations are shown in fig. 6 and fig. 7.

It can be observed that there is practically no difference in the stability regions obtained with one and six precursor group calculations (for both calculations – with and without xenon). In fact, in two cases, the coefficients of the limiting inequalities (that determine the stability boundary) differ only after several significant digits. It is clear that one group approximation can be used for the linear stability analysis without practically affecting the results (as compared to six group calculations) while considerably simplifying the calculations. Further, it was found that in addition to the number of precursor groups considered, the variations in the kinetic parameters (*i. e.* $\Lambda, \lambda_i, \beta_i$) also have insignificant influence on the location of the stability boundary. However, location of stability boundary is largely influenced by the parameters governing the evolution of fuel and coolant temperature and xenon

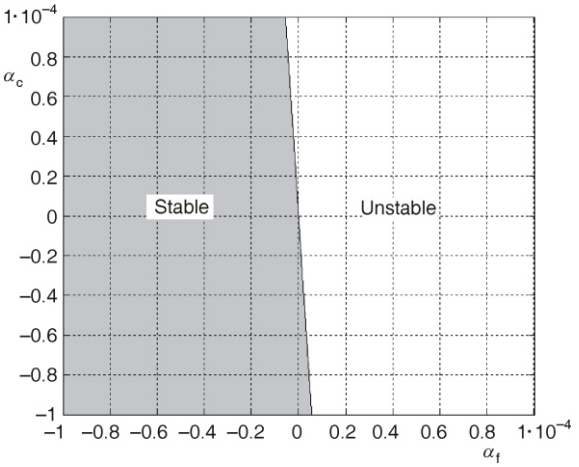

Figure 6. Stability map without Xe (*i* = 6 groups)

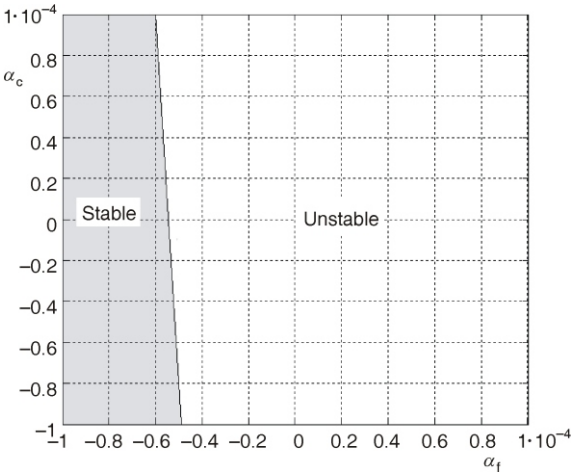

Figure 7. Stability map with Xe (*i* = 6 groups)

concentration (*i. e.* the coefficients $a_1, a_2, \ldots . a_7$ in eqs. (21), (22), (30), and (31)). It must be mentioned here that the evolution of power transient following the reactivity change is very sensitive to the kinetic parameters. However, as the present analysis indicates, they have a negligible influence on inherent stability of the reactor around the equilibrium point (based on the linearized system) and it is mainly dependent on the feedback parameters (see Appendix).

Reactor behaviour at a typical operating point

The reactivity coefficients change with the core burn-up over the operating period of the reactor. The evolution of the reactor operating point ($\alpha_{f,op}$, $\alpha_{c,op}$) with burn-up and corresponding stability margins can be traced using such stability maps (stability boundary itself may change slightly with burn-up). A typical operating point for PHWR is shown in fig. 8. It is clear that at this operating point, the reactor is unstable due to the xenon reactivity effect.

The Pole-Zero maps for the overall transfer function – closed loop transfer function, eq. (45) – at this operating point without and with xenon reactivity effect are shown in figs. 9 and 10, respectively. It is observed that when xenon effect is neglected, all the poles/eigenvalues are in the left half of the s-plane (the reactor is stable). When xenon effect is included, there are two poles in the positive half and the reactor is unstable. The same has also been verified using the Routh-Hurwitz criterion.

The xenon feedback effect is slower in time as compared to other reactivity feedbacks. Hence, the xenon reactivity effects are usually neglected while simulating power transients of short duration. However, it is clear that the xenon affects the inherent stability of the reactor and needs to be taken into account while designing the reactivity control measures/equipment. A reliable external reactivity control system (reactor regulating system of PHWR) is required to maintain the reactor at steady operating condition.

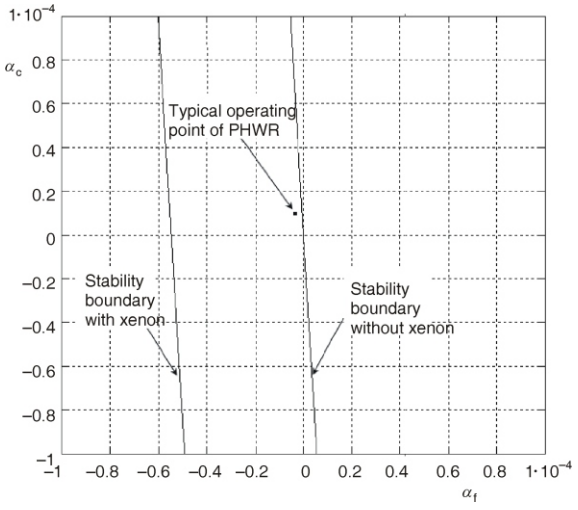

Figure 8. Typical operating point of PHWR on the stability map

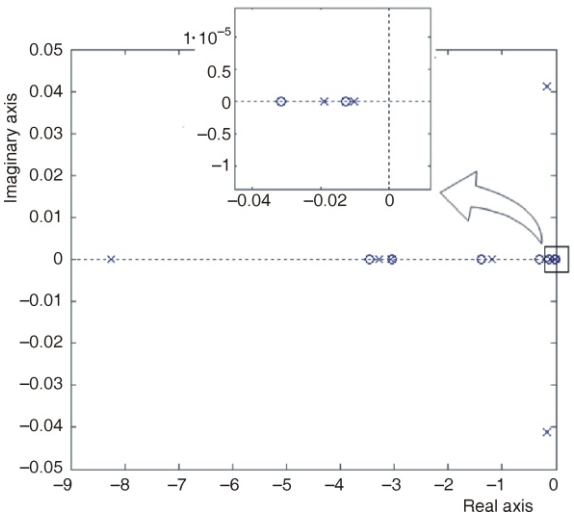

Figure 9. Pole-Zero map (without xenon effect)

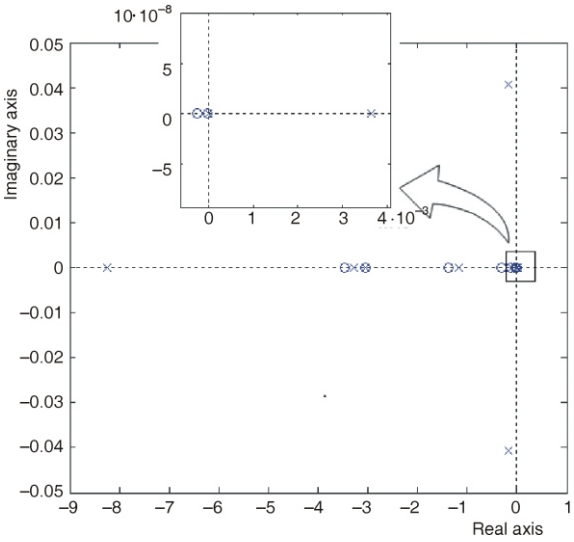

Figure 10. Pole-Zero map (with xenon effect)

CONCLUSIONS

A methodology has been established for carrying out the linear stability analysis of nuclear reactors using the lumped reactor model including the reactivity feedbacks from the fuel and the coolant temperature and xenon. The non-linear terms involved in the set of coupled governing equations were removed by considering the small perturbations in the variables around their steady-state values. The set of linear equations thus obtained was used for the stability analysis through transfer function as well as eigenvalue approach. The methodology established here was used for carrying out the linear stability anal-

ysis of a typical PHWR. The stability maps (the region of stability in terms of fuel and coolant temperature reactivity coefficients) were obtained for this reactor. It was shown that the positive reactivity feedback from xenon narrows down the region of stability. Both six and one precursor group point kinetics equations were used to find the stability maps separately. No significant differences were observed in two cases. This indicates that equivalent one precursor group equations can be used to obtain the accurate results without elaborate six group calculations. Further, it was found that when the reactivity coefficients are considered as parameters, the stability of the reactor is largely influenced by the factors governing the evolution of fuel and coolant temperature and xenon concentration rather than the neutron kinetics parameters.

The method of analysis presented in this paper is limited in its applicability to the behaviour of reactor in response to small deviations from the steady-state. The response of reactor to finite reactivity perturbations can be obtained by actually solving the reactor dynamics equations. Nevertheless, the technique used here provides valuable insights into the neutronic stability of reactors around a given equilibrium point by appropriately considering different reactivity feedback mechanisms. Further, the methodology discussed here is applicable to the reactors that use single phase coolant (*e. g*. PWR, PHWR), since void reactivity effects have not been modelled. However, it can be extended to the reactors with two phase coolant (*e. g*. BWR) by including the void reactivity feedback through the appropriate time dependent formulation.

APPENDIX

As mentioned in the section *Determining the Stability of Reactor*, the stability map is the region of intersection of the inequalities obtained from the terms of the characteristic equation. For the case with one precursor group and no xenon feedback, the inequality that determines the stability boundary (see eq. (51)) is

$$7.91\ 10^{2}\alpha_{\mathrm{f}} \quad 4.40\ 10^{3}\alpha_{\mathrm{c}} \quad 0 \qquad \text{(A1)}$$

Expressed explicitly in terms of parameters, (A1) becomes*

$$(a_3 \quad a_4)\alpha_{\mathrm{f}} \quad a_3\alpha_{\mathrm{c}} \quad 0 \qquad \text{(A2)}$$

It is seen that in this case, there is no dependence on the kinetics parameters $\beta, \lambda,$ or Λ.

In the case of one precursor group and xenon feedback included, the inequality that determines the stability boundary (see eq. (52)), is

$$2.14\ 10^{5}\alpha_{\mathrm{f}} \quad 1.19\ 10^{6}\alpha_{\mathrm{c}} \quad 1.15\ 10^{9} \quad 0 \qquad \text{(A3)}$$

Expressed in terms of the kinetic parameters, (A3) becomes (values of $P_0, \rho_{X0}, a_1, a_2, \ldots, a_7$ are substituted to simplify the expression)

$$\frac{1.43\ 10^{15}}{\lambda} \quad 5.47\ 10^{11}\ \alpha_{\mathrm{f}} + \frac{7.96\ 10^{17}}{\lambda} \quad 3.04\ 10^{12}\ \alpha_{\mathrm{c}} \quad 3.8\ 10^{18}\Lambda \quad \frac{3.8\ 10^{18}\beta}{\lambda} \quad \frac{1.45\ 10^{20}}{\lambda} \quad 2.95\ 10^{15} \quad 0 \qquad \text{(A4)}$$

It is clear that for the typical range of values of the kinetics parameters β, λ and Λ, the relative magnitudes of the terms involving the kinetic parameters are much smaller than the other terms which are independent of them. Example in (A4), in the coefficient of α_{f}, for the typical values of λ, the term $1.43\ 10^{-15}/\lambda$ is a couple of orders of magnitude smaller than the term $5.47\ 10^{-11}$.

Similarly, for the system with six precursor groups (both with and without xenon), it can be shown that the terms involving $\beta_{\mathrm{i}}, \lambda_{\mathrm{i}}$, and Λ are much smaller in magnitude than the terms independent of them. Hence, it is clear that the location of the stability boundary (*i. e*. its slope and intercept) is not significantly influenced by the point kinetics parameters. It is mainly determined by the thermal-hydraulic and xenon dynamics parameters.

AUTHORS' CONTRIBUTIONS

V. A. Kale, R. Kumar, and K. Obaidurrahman are working on the development and the validation of reactor dynamics models at the Atomic Energy Regulatory Board for safety analysis of Indian NPP. The present work has been carried out by V. A. Kale and R. Kumar under the guidance of K. Obaidurrahman. A. J. Gaikwad, Director of Nuclear Safety Analysis Division, has been leading a team of engineers in nuclear safety analysis and research at Atomic Energy Regulatory Board India.

NOMENCLATURE

A – area, [m^2]
C – concentration, [m^{-3}], or specific heat, [$Jkg^{-1}\,°C^{-1}$]
h – heat transfer coefficient, [$Wm^{-2}\,°C^{-1}$]
m – mass, [kg]
$\dot{m}$ – mass flow rate, [kgs^{-1}]
P – power, [W]
T – temperature, [°C]
t – time, [s]
x – non-dimensional power change
y – non-dimensional concentration change
z – normalized reactivity or temperature

Greek symbols

α – reactivity coefficient, [$°C^{-1}$]

* Inequalities (A1) and (A2) differ only by a constant positive multiplying factor. The same is true for (A3) and (A4)

β – delayed neutron fraction
γ – fractional fission yield
Λ – neutron generation time, [s]
λ – decay constant, [s^{-1}]
ρ – reactivity
Σ – macroscopic cross-section, [m^{-1}]
σ – microscopic cross-section, [m^2]
ϕ – neutron flux, [$m^{-2}s^{-1}$]

Subscripts

c – coolant
I – iodine
i – i-th delayed neutron precursor group
f – fuel
X – xenon
0 steady-state value

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ЛИНЕАРНА АНАЛИЗА СТАБИЛНОСТИ НУКЛЕАРНОГ РЕАКТОРА КОРИШЋЕЊЕМ КОНДЕНЗОВАНОГ МОДЕЛА

У пројектовању и управљању нуклеарним реактором анализа стабилности реактора значајно је питање, јер је са гледишта сигурности постојан неутронски одзив на поремећај од суштинске важности. У овом раду, развијена је општа методологија линеарне анализе стабилности нуклеарних реактора коришћењем кондензованог реакторског модела. Кинетика реактора моделована је коришћењем једначина тачкасте кинетике и повратне спреге са горивом, док су хладилац и ксенон моделовани одговарајућим временски зависним једначинама. Ове управљачке једначине линеаризоване су при разматрању малих поремећаја стања реактора око стабилне радне тачке. Карактеристична једначина система користи се за успостављање зоне стабилности реактора с обзиром на коефицијенте реактивности као параметре. Ова методологија употребљена је за идентификовање области стабилности типичног PHWR реактора. Показало се да позитивна повратна спрега реактивности од ксенона сужава област стабилности. Даље је уочено да неутронски кинетички параметри (као што су број група закаснелих неутрона, време генерисања неутрона, фракције закаснелих неутрона, итд.) немају значајан утицај на положај границе стабилности. Граница стабилности у великој мери зависи од параметара који регулишу промену температуре горива и хладиоца и концентрацију ксенона.

Кључне речи: линеарна стабилност, PHWR, коефицијент реактивности, ксенонска повратна спрега, област стабилности