ANALYSIS OF CORRELATION AND REGRESSION BETWEEN PARTICLE IONIZING RADIATION PARAMETERS AND THE STABILITY CHARACTERISTICS OF IRRADIATED MONOCRYSTALLINE Si FILM

by

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This paper deals with the analysis of correlation and regression between the parameters of particle ionizing radiation and the stability characteristics of the irradiated monocrystalline silicon film. Based on the presented theoretical model of correlation and linear regression between two random variables, numeric and real experiments were performed. In the numeric experiment, a simulation of the effect of alpha radiation on a thin layer of monocrystalline silicon was performed by observing a number of vacancies along the film depth resulting from a single incident alpha particle. In the real experiment, the irradiation of a thin silicon film by alpha particles from a radioactive Am-241 alpha emitter was performed. The observed values of radiation effect on the Si film were specific resistance and the concentration of free charge carriers. The results showed a fine concordance between numeric and real experiments. Correlation verification of the observed values was presented by linear regression functions.

Key words: alpha particle, Si film, vacancy, specific resistance, free charge carrier, correlation, regression

INTRODUCTION

The usage of modern electronic devices in the conditions of ionizing radiation (nuclear technology, military technology, space technology) imposes the need to examine their radiation hardness, that is, their reliability in such application conditions. Increasing trend towards the miniaturization of electronic components additionally actualizes this issue so in recent times there have been a considerable number of scientific papers in this area [1-5]. However, a common deficiency of all those papers is that the results are presented either qualitatively or statistically. This paper examines a possibility of expressing radiation hardness by a single-valued, quantitative indicator, the correlation coefficient, and regression lines.

If several random values are measured simultaneously on the examination samples, then we want to know whether these values are interrelated, how strong their relationship is and how it can be expressed mathematically. Correlation and regression provide a solution to those problems [6].

Correlation analysis first deals with the issue whether there are any linear dependencies between the random values X and Y that are examined, and how strong they are, according to the correlation coefficient r. It is assumed that the random variables X and Y have a normal distribution. Correlation coefficient can have $|r|$ 1values. X and Y are non-correlated for $r = 0$, *i. e.*, there is no linear dependence between them. The closer is to one, the stronger is the correlation between these values. When $r > 0$, X and Y increase or decrease together: this is a "positive correlation". When $r < 0$, large values of Y are related to the small values of X: this is a "negative correlation". Value $|r|$ 1 represents a complete correlation, *i. e.*, a perfect functional dependence.

The aim of this paper is to establish the relation between the microscopic quantity of a number of vacancies per unit of depth per incident particle and the macroscopic quantities' specific resistance and the concentration of free charge carriers in order to examine the radiation hardness of monocrystaline Si thin films.

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CORRELATION AND REGRESSION

Empirical correlation coefficient can be determined in the following way: let us consider a sample consisting of the n pairs of values (x_i, y_i). They will be determined by arithmetic mean values $\bar{x}$ and $\bar{y}$ and mean square deviations s_x and s_y. Further, empirical covariance is introduced [7]

$$s_{xy} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}) \tag{1}$$

which relates the two values, and represents the product of mean deviations.

The empirical correlation coefficient is obtained from the following quantities

$$r = \frac{s_{xy}}{s_x s_y} \tag{2}$$

this can also be calculated through [7]

$$r = \frac{\sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}}{\sqrt{\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right)\left(\sum_{i=1}^{n} y_i^2 - n\bar{y}^2\right)}} \tag{3}$$

Regression analysis, based on samples, examines, a possibility of the existence of a functional relationship between random variables X and Y, on the one hand, and, a random variable and parameters, on the other hand. Two problems with different contents have been mathematically treated in the same way. In the simplest case, pairs of values (x_i, y_i) would be graphically displayed for regression (x_i would then, for instance, be the parameter, and y_i would be the realised random variable). Here we will only consider an instance when there is a linear dependence between the values of X and Y.

In the linear regression, assuming that the random variables are normally distributed, random values are mostly related by the linear function. This linear regression line represents a mathematical expectation

$$EY = \alpha + \beta EX \text{ i.e.}, Y = \alpha + \beta X \tag{4}$$

In the simplest case, it is graphically derived from the samples by drawing the optimal line through graphically represented pairs of values (x_i, y_i), fig. 1. If accurate values are required, the estimated values must be calculated for coefficients α and β by the least square deviation method.

If the outcomes are very dispersed on the regression line and if accordingly, the direction coefficient is small, the linear dependence between the quantities of X and Y is weak, fig. 1(a). If, on the other hand, dispersion is weaker, and β large, it indicates a strong dependence, fig. 1(b).

It is important to make a correct distinction between dependent and independent quantities (parame-

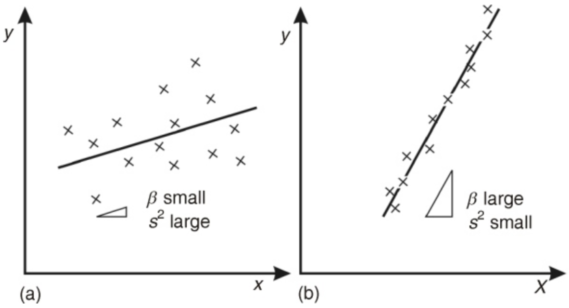

Figure 1. Position of the outcome of quantities *X* and *Y*, as a function of the strength of their dependence (schematically): (a) weak dependence, (b) strong dependence; β – direction coefficient, s^2 – variance

ters). In estimating y from x, the empirical regression line

$$y = a_{yx} + b_{yx}x \tag{5}$$

represents an optimal fitting of the empirically defined values. The sum of vertical deviations is minimal, whereas reverse case

$$x = a_{xy} + b_{xy}y \tag{6}$$

the sum of horizontal deviations is minimal.

As the correlation increases, the distinction between regression lines described by the eqs. (5) and (6) disappears. For correlation coefficient $|r| = 1$, regression lines coincide.

Corresponding regression lines, for the same sample, are determined in the following way. In the regression y from x, direction coefficient is calculated as

$$b_{yx} = \frac{s_{xy}}{s_x^2} = r\frac{s_y}{s_x} \tag{7}$$

$$b_{yx} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2} \tag{8}$$

Thus, for the regression x from y the following is valid

$$b_{xy} = \frac{s_{xy}}{s_y^2} = r\frac{s_x}{s_y} \tag{9}$$

$$b_{xy} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(y_i - \bar{y})^2} \tag{10}$$

A free element of regression line a for two cases is obtained as

$$a_{yx} = \bar{y} - b_{yx}\bar{x} \tag{11}$$

$$a_{xy} = \bar{x} - b_{xy}\bar{y} \tag{12}$$

THE EXPERIMENT AND THE PROCESSING OF EXPERIMENT RESULTS

In order to verify the method of numeric simulation of the interaction of bundles of heavy ions with semiconducting materials, and their consequences, numeric experiments and, simultaneously, the corresponding real experiments were conducted.

For numeric simulation of the radiation effect on the thin layer of monocrystalline silicon, Monte Carlo method was used. The simulation of the heavy ions *i. e.*, alpha particles of the energy of 5.5 MeV, passing, through a thin layer of monocrystalline Silicon was conducted in the TRIM Module of the SRIM software package [8].This software provides calculation of the energy loss of incident radiation through ionization, phonon stimulation of the grid and the displacement of the atoms of the material. Bundles of alpha particles are selected in a way that they correspond to the conditions of the real experiment. Simulation result is expressed by the number of the resulting vacancies in monocrystalline silicon.

The real experiment was conducted in a way that a thin layer of monocrystalline silicon of the *n*-type was irradiated with 10^3 alpha particles of 5.5 MeV energy from a radioactive Am-241 alpha emitter. On such irradiated sample, by the four-point method, the concentration of free charge carriers and the specific resistance was determined in 50 points. The extended combined measurement uncertainty of the experiment procedure was less than 5% [9, 10].

Results obtained by numeric and real experiment were processed in a way that the following has been determined: 1 – the coefficient of correlation between the concentration of free charge carriers and the specific resistance, according to the eq. (3), 2 – the coefficient of correlation between the concentration of free charge carriers and the number of vacancies resulting from the effect of alpha radiation, according to the eq. (3), 3 – the coefficient of correlation between the specific resistance and the number of vacancies resulting from the effect of alpha radiation, according to the eq. (3), 4 – the regression of the number of free charge carriers to the number of vacancies resulting from the effect of alpha radiation, according to the eqs. (5) to (12), and 5 – the regression of the specific resistance to the number of vacancies resulting from the effect of alpha radiation, according to the eqs. (5) to (12).

RESULTS AND DISCUSSION

Figure 2(a) displays the number of vacancies in the silicon per micrometre (observed along the depth inside the film) per single incident alpha particle determined by the numeric simulations under following conditions: the total number of simulations is 50; the number of alpha particles in the incident bundle per each simulation is 10^3; the energy of alpha particles is 5.5 MeV, the thickness of the monocrystalline silicon film is 35 μm. Figure 3(b) displays the results of 4 different numeric calculations displayed in a single graph.

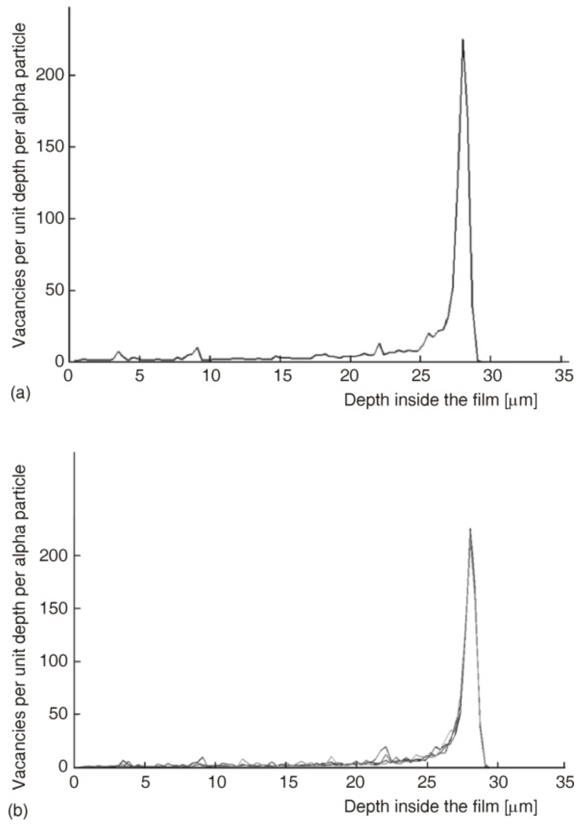

Figure 2. The number of vacancies per single unit of depth inside the film, including the vacancies produced by the displaced atoms of Si: the results of a single numeric simulation (a), and the results of 4 simulations displayed in a single graph (b)

Algebraic value of the correlation coefficient between the statistic sample of random variable "specific resistance" and statistic sample of random variable "concentration of free charge carriers" is 1.

Such result was expected since the random variables are theoretically related by the linear dependence at the temperature at which the experiment was performed. Further, if the correlation coefficient equals 1, it can be claimed that the real experiment was verified. The value of the correlation coefficient between the statistic sample of random variable "the number of vacancies per incident alpha particle" and corresponding statistic sample of random variable "specific resistance" is 0.9706; the value of correlation coefficient between statistic sample of random variable "number of vacancies per alpha particle" and corresponding statistic sample of random variable "concentration of free charge carriers" is –0.9706.

Such values of the correlation coefficients between the result of numeric and real experiment completely verify numeric and experiment procedures. Physically observed algebraic sign of the correlation

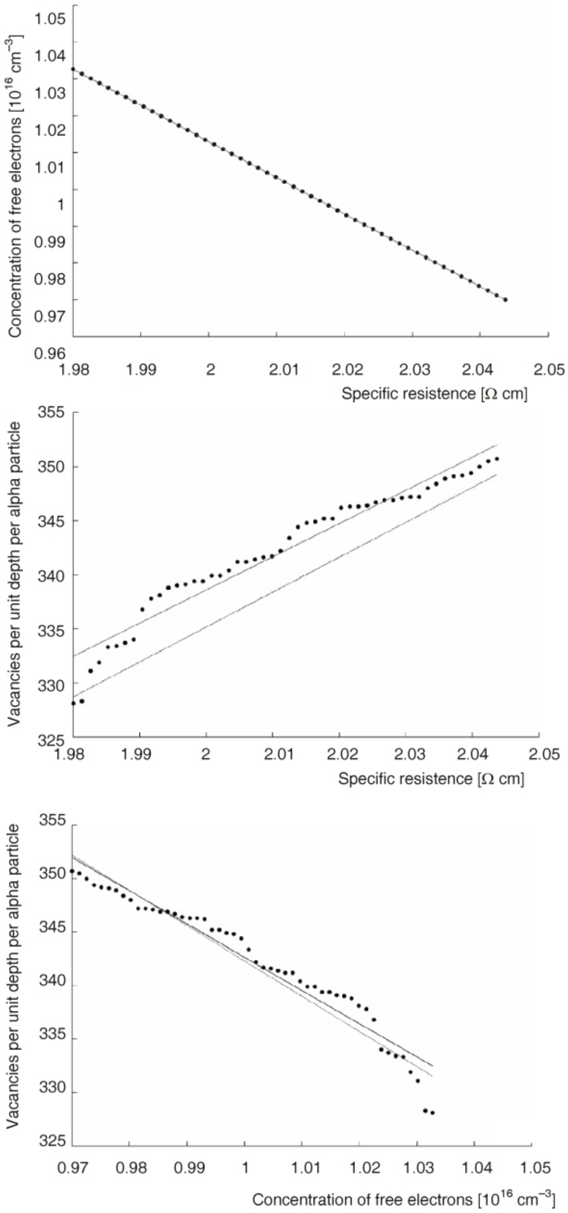

Figure 3. Linear regression curves: the concentration of free charge carriers *vs*. specific resistance (a), the number of vacancies per single unit of the depth inside the film per single incident alpha particle *vs*. specific resistance (b), and the number of vacancies per single unit of the depth inside the film per single incident alpha particle *vs*. the concentration of free charge carriers (c)

coefficients is expected since the vacancies "suck in" free charge carriers and thus reduce their concentration [11], which linearly reduces the corresponding values of specific resistance, fig. 3(a). The results were also confirmed by the linear regression functions displayed in the figs. 3(b) and 3(c).

CONCLUSIONS

According to the results obtained from the numeric and real experiments, a possibility of expressing radiation hardness of the material by a single-valued, quantitative indicator, the correlation coefficient is presented in the paper. Applying linear regression to establish a relation between the microscopic quantity of a number of vacancies per unit of depth per incident particle and the macroscopic quantities specific resistance and the concentration of free charge carriers, an original approach in the examination of the radiation hardness of the material was presented.

In the radiation environment where the characteristics of the material are observed mainly through stochastic quantities, the regression method represents a good approach in the prediction of the stability of the characteristics of the material and the components used in hostile, *i. e*., extreme working conditions.

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AUTHOR CONTIBUTIONS

Theoretical analysis was carried out by U. G. Jakšić and N. B. Arsić. Experiments were carried out by U. G. Jakšić and K. Dj. Stanković. All of the authors have analysed and discussed the results. The manuscript was written by U. G. Jakšić. The figures were prepared by K. Dj. Stanković.

REFERENCES

[1] Srour, J. R., Marshall, C. J., Marshall, P. W., Review of Displacement Damage Effects in Silicon Devices, *IEEE Transactions on Nuclear Science, 50* (2003), 3, pp. 653-670

[2] Pejović, M. M., *et al*., Successive Gamma-Ray Irradiation and Corresponding Post-Irradiation Annealing of pMOS Dosimeters, *Nucl Technol Radiat, 27* (2012), 4, pp. 341-345

[3] Vujisić, M., *et al*., Radiation Hardness of COTS EPROM and EEPROM, *Radiation Effects and Defects in Solids: Incorporating Plasma Science and Plasma Technology, 165* (2010), 5, pp. 362-369

[4] Vujisić, M., *et al*., Influence of Radiation on Titanium Dioxide Mermistors, *Scientific Publications of the State University of Novi Pazar Series A: Applied Mathematics, Informatics & Mechanics, 4* (2012), 1, pp. 75-82

[5] Lazarević, Dj., *et al*., Radiation Hardness of Indium Oxide Films in the Cooper-Pair Insulator State, *Nucl Technol Radiat, 27* (2012), 1, pp. 40-43

[6] Hill, T., Statistical Mechanics, Principles and Selected Applications, McGraw-Hill Book Company, Inc, 1956

[7] Hauschild, W., Mosch, W., Statistical Techniques for High-Voltage Engineering, Peter Peregrinus Ltd, London, 1992

[8] Ziegler, J. F., Biersack, J. P., Ziegler, M. D., SRIM (The Stopping and Range of Ions in Matter), http://www.srim.org

[9] Vujisić, M., Stanković, K., Osmokrović, P., Statistical Analysis of the Characteristics of Some Basic Mass-Produced Passive Electrical Circuits Used in Measurements, *Measurement, 44* (2011), 9, pp. 1713-1722

[10] Kovačević, A., *et al*., Uncertainty Evaluation of the Conducted Emission Measurements, *Nucl Technol Radiat, 28* (2013), 2, pp. 182-190

[11] Holmes-Siedle, Adams, L., Handbook of Radiation Effects, Oxford Science publication, Oxford, UK, 1993

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АНАЛИЗА КОРЕЛАЦИЈЕ И РЕГРЕСИЈЕ ИЗМЕЂУ ПАРАМЕТАРА ЧЕСТИЧНОГ ЈОНИЗУЈУЋЕГ ЗРАЧЕЊА И СТАБИЛНОСТИ КАРАКТЕРИСТИКА ОЗРАЧЕНОГ МОНОКРИСТАЛНОГ Si ФИЛМА

У овом раду анализиране су корелација и регресија између параметара честичног јонизујућег зрачења и стабилност карактеристика озраченог монокристалног силицијумског филма. На основу приказаног теоријског модела корелације и линеарне регресије између две случајне променљиве извршени су нумерички и реални експерименти. У нумеричком експерименту симулирано је дејство алфа зрачења на танки слој монокристалног силицијума, посматрајући број шупљина по дубини филма насталих по једној инцидентној алфа честици. У реалном експерименту озрачен је танак силицијумски филм алфа честицама из радиоактивног алфа емитера Am-241. Посматране величине дејства зрачења на Si филм биле су специфична отпорност и концентрација слободних носилаца наелектрисања. Резултати су показали добро слагање између нумеричког и реалног експеримента. Верификација корелације посматраних величина приказана је линеарним регресионим функцијама.

Кључне речи: алфа честица, Si филм, шупљина, специфична отпорност, корелација, регресија, слободни носилац наелектрисања