GETTING DEEPER INSIGHT INTO STOPPING POWER PROBLEMS IN RADIATION PHYSICS USING THE NOETHER'S THEOREM COROLLARY

by

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The theories that combine two different approaches in dealing with interacting objects, for instance, treating electromagnetic laser field classically, and the interacting atom as a quantum object, have some ambiguities and, as such, they should be labeled as "mixed". From the Noether's Theorem Corollary, which we proved earlier, about the conservation laws of energy, momentum and angular momentum in mixed theories, follows that the aforementioned theories do not support the law of angular momentum/spin conservation (to be precise, the obtained result does not imply that the law of conservation of angular momentum and spin is not valid generally, but rather that mixed theories can produce the results which might violate this law). In present paper, an additional explanation following our Corollary is given to why the calculation of the stopping power in the fully quantized theory gives better results than those that were obtained in mixed theories, which further confirms the predictions of our Corollary.

Key words: stopping power, quantum harmonic oscillator, Noether's theorem, mixed theory, conservation law

INTRODUCTION

In earlier papers [1, 2] it was emphasized that theories describing the behavior of atoms in strong laser fields treat the atom as a quantized object and that the electromagnetic field is treated classically. We have already suggested that these theories should be called "mixed". Although using mixed theories in the beginning of the Quantum Theory was considered unpleasant [2], these theories (which combine the classical and the quantum approach) have shown their vitality (for instance, the ADK theory [3]) not only in the case of strong laser fields but also in case of super-strong fields. Also, in calculating the stopping power in Radiation Physics until very recently the situation could have been described in a similar manner: a projectile atom was considered a point-like object pretty much as in the classical case, and the rest of the theory was based on the first Born approximation which is a fully quantum approach [4]. Sigmund and Haagerup [5] also treated the problem using the classical and the quantum approach (again, a "mixed" theory) – the projectile was treated as a classical charged particle and the target as a quantum harmonic oscillator. Later, Cabrera-Trujillo [6], based on [5], included, more thoroughly (but not completely), the quantum mechanical approach. However, Stevanović and Nikezić [7, 8] felt the need to treat both projectile and target as a set of quantum harmonic oscillators, thus improving the procedure for a more precise calculation of the stopping power.

THE COROLLARY TO NOETHER'S THEOREM

In order to shed some new light on the aforementioned results, we shall concisely present some of our earlier arguments [2], and prove the Corollary to Noether's theorem about the behavior of mixed theories when they treat the angular momentum and spin.

In the following paper [2], we could thus rephrase the formulation of Noether's theorem: To any s-parametric continuous transformation of field functions and co-ordinates, which keeps the variation of action zero, there are corresponding s-dynamic invariants (*i. e.* a constant in time combinations of field functions and their derivatives).

In what follows, the Greek indices are denoting 4-co-ordinates of space-time (0-3) while the Latin

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ones are denoting the spatial 3-co-ordinates (1-3). The starting point is the expression (11) from [9], adjusted to our present needs

$$\theta_{\nu}^{\kappa} \quad \frac{\partial L}{\partial u_{\alpha;\kappa}}(u_{\alpha;\lambda} X_{\nu}^{\lambda} \quad \psi_{\alpha\nu}) \quad L(x) X_{\nu}^{\kappa} \tag{1}$$

which describes the tensor quantity that appears in the action integral obtained for the s-parametric transformations (here $L(x)$ is the Lagrangian of the system, and covariant derivative is denoted by ";"). The usual method is to introduce an infinitesimal transformation of 4-coordinates, for which the infinitesimal continuous space-time translations are chosen

$$x^{\kappa} \quad x^{\kappa} \quad x^{\kappa} \quad \delta x^{\kappa} \tag{2}$$

while, for the transformation of field functions, one has

$$u_{\alpha}(x) \quad u_{\alpha}(x\ ) \quad u_{\alpha}(x) \quad \delta u_{\alpha}(x) \tag{3}$$

The variations δx^{κ} and δu_{α} are expressed using the infinitesimal linearly independent parameters of the transformation $\delta\omega^{\nu}$

$$\delta x^{\kappa} \quad \underset{1\ \nu\ s}{} X_{\nu}^{\kappa} \delta\omega^{\nu}, \quad \delta u_{\alpha}(x) \quad \underset{1\ \nu\ s}{} \Psi_{\alpha\nu} \delta\omega^{\nu} \tag{4}$$

where s represents the number of parameters of transformation, and is not restricted by our confining Latin indices to three dimensions, *i. e.* it could be a number greater than three; $\delta\omega^{\nu}$ are parameters themselves.

Now, by returning to the expression (2), choosing for parameters of transformation the infinitesimal translations δx^{κ}, one obtains, from (4)

$$X_{\lambda}^{\kappa} \quad \delta_{\lambda}^{\kappa}, \quad \Psi_{\alpha\lambda} \quad 0 \tag{5}$$

Then follows that the tensor θ_{ν}^{κ} from the eq. (1) becomes a mixed second rank tensor

$$T_{\nu}^{\kappa} \quad \frac{\partial L}{\partial u_{\alpha;\kappa}(x)} \frac{\partial u_{\alpha}}{\partial x^{\nu}} \quad L\delta_{\nu}^{\kappa} \tag{6}$$

or, written in its fully contra variant form

$$T^{\lambda\kappa} \quad \frac{\partial L}{\partial u_{\alpha;\kappa}(x)} \frac{\partial u_{\alpha}}{\partial x^{\lambda}} \quad L g^{\kappa\lambda} \tag{7}$$

In [9, 10] it shown that integrals over the three-dimensional configuration space, more precisely, the integrals of the type

$$C_{\nu}(x^{0}\ ) \quad \theta_{\nu}^{0} \mathrm{d}\vec{x} \tag{8}$$

are constant in time.

For the zero component of tensor $T^{\lambda\kappa}$ (7), such an integral would give a constant in time 4-vector

$$P^{\lambda} \quad T^{\lambda 0} \mathrm{d}\vec{x} \tag{9}$$

A zero component of this vector is, in fact, a Hamilton function of classical mechanics, *i. e.* the energy.

As time and space translations (which are involved in transformation (2) that is responsible for the resulting conservation laws) were not quantized in all physical theories (*i. e.* as time and space are homogenous in the same manner both in classical and quantum case), the part of the theory which uses the classical approach can be smoothly connected with the one that uses the quantum approach. Therefore, the law of conservation of momentum/energy is obtained. In order to separate the energy part from the momentum part, only the double zero component T^{00} of tensor (7) could be taken

$$E \quad P^{0} \quad T^{00} \mathrm{d}\vec{x} \quad \text{const.} \tag{10}$$

obtaining thus the conservation law of energy alone, which is the result of the time translation. As the time is homogenous in the same manner in both the classical and quantum case, the expression (10) is applicable to mixed theories.

Now, the infinitesimal Lorentz rotations will be introduced into the eq. (2): $\delta x^{\kappa} \quad \delta L_{\nu\mu}$, where $\delta L_{\nu\mu}$ are the infinitesimal parameters of the rotations; since they are anti symmetrical and independent parameters six of them can be chosen in total

$$\delta\omega_{(\kappa\mu)} \quad \delta L_{\mu\nu}, \quad (\mu \quad \nu) \tag{11}$$

which represent the infinitesimal angles of the rotation in the $x_{\mu}x_{\nu}$ plane.

After some calculations, the following expressions, as per [2, 8], are obtained

$$X_{\nu}^{(\rho\sigma)} \quad x^{\sigma}\delta_{\nu}^{\rho} \quad x^{\rho}\delta_{\nu}^{\sigma} \tag{12}$$

and also

$$\Psi_{\mu}^{\nu(\rho\sigma)} \quad A_{\mu}^{\nu(\rho\sigma)} u_{\mu}(x) \tag{13}$$

where the quantity $A_{\mu}^{\nu(\rho\sigma)}$ is given, for vector fields, as

$$A_{\mu}^{\nu(\rho\sigma)} \quad \delta_{\mu}^{\rho} g^{\nu\sigma} \quad \delta_{\mu}^{\sigma} g^{\nu\rho}, \quad (\rho \quad \sigma) \tag{14}$$

In accordance with eqs. (12)-(14), the expression for 4-angular momentum tensor follows

$$\begin{aligned} M^{\tau(\rho\sigma)} \quad & \frac{\partial L}{\partial u_{\nu;\tau}}(u_{\nu}^{\tau;\rho} x^{\sigma} \quad u_{\nu}^{\tau;\sigma} x^{\rho}) \\ L(x^{\rho} g^{\sigma\tau} \quad & x^{\sigma} g^{\rho\tau}) \quad \frac{\partial L}{\partial u_{\nu;\tau}} A_{\mu}^{\nu(\rho\sigma)} u_{\mu}(x\ ) \\ (x^{\sigma} T^{\rho\tau} \quad & x^{\rho} T^{\sigma\tau}) \quad \frac{\partial L}{\partial u_{\nu;\tau}} A_{\mu}^{\nu(\rho\sigma)} u_{\mu}(x\ ) \end{aligned} \tag{15}$$

The first term in (15) can be represented as

$$M_{0}^{\tau(\rho\sigma)} \quad x^{\sigma} T^{\rho\tau} \quad x^{\rho} T^{\sigma\tau} \tag{16}$$

which is easily identified as an orbital angular momentum of the wave field, while the second part is denoted in the following manner

$$S^{\tau(\rho\sigma)} \quad \frac{\partial L}{\partial u_{a;\tau}} A_{a}^{b(\rho\sigma)} u_{b} \tag{17}$$

and it characterizes the polarization properties of the field in the case of the multi component fields, when the last term in eq. (15) is different from zero. In the quantum case (17) corresponds to the spin of the particle as described by the quantized field, as per [2, 9].

When the 4-rotations of space-time are chosen, they result in the conservation of 4-angular momentum (*i. e.* 3-angular momentum and spin), but this is not applicable to mixed theories, as the rotations, which are continuous in the classical theory, have to be quantized in the quantum theory[1]. This changes the type of continuity of parameters, since in the classical case, one uses continuous functions, while in the quantum case, operators (the counterparts to functions) are used, and they are not continuous, but closed. The closeness [2, 10] is the quality of operators in Hilbert spaces, which is necessary in order to use them in the quantum mechanical formalism, if they are not bounded, and thus not continuous. This is the case with most of the fundamental quantum mechanical operators, such as the angular momentum operator, so in that sense the closeness of an angular momentum operator (as a parameter in Noether's theorem) changes its type of continuity as compared to the parameters based on classical functions. It can be said that the isotropy of space is somehow broken (indicating that space is not isotropic in mixed theories). Consequently, there can be no smooth connection between the classical and quantum part of the theory, hence for such theories the conservation of angular momentum and spin is not working.

In this manner was proven the following [1, 2].

Noether's theorem corollary

Physical theories that combine ("mix") quantum and classical approach do support the law of energy/momentum conservation, but do not support the law of angular momentum/spin conservation.

It should be stressed out that this proof, being all-inclusive, holds for all mixed theories.

The aforementioned result does not imply that the law of conservation of angular momentum/spin is not valid generally, but rather that mixed theories are producing results which might contradict this law. This shortcoming of mixed theories is not so often evident since most of these theories do not deal with angular momentum/spin (for instance, the ADK-theory, explaining the process of tunneling ionization of atoms by strong laser fields). At this point we are weakening our argument, as it is not needed to rule out the whole mixed theory, because it fails to support one specific law of conservation. Thus the suggestion to the scientists that use mixed theories in their researches is to carefully check the part of their results referring to the angular momentum/spin.

[1]As rotations involve time (in order to obtain spin) one has a different situtiion in the classical and quantum case [9]

DISCUSSING THE RESULTS OF STEVANOVIĆ AND NIKEZIĆ

In the fig. 1, taken from [7], the results of the Bethe theory [4], the Carera-Trujillo theory [6] and the Stevanović-Nikezić theory [7] are shown, together with the SRIM (The stopping and range of ions in matter – the group of programs for calculating the stopping power and range in matter for ions up to 2 GeV per nucleus), which is based on the SRIM simulation of experimental data.

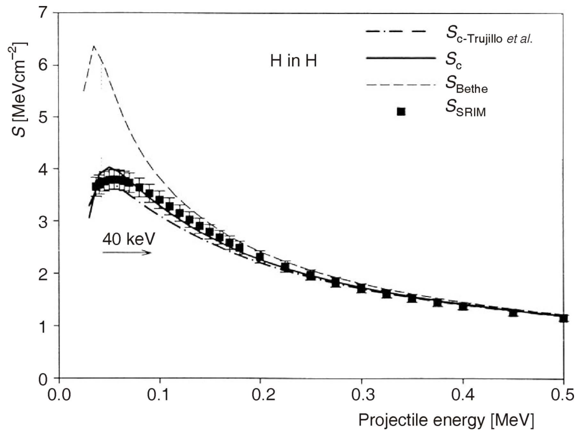

Figure 1. The stopping power of hydrogen for hydrogen ion as calculated per Bethe's, Cabrera-Trujillo's, and Stevanović-Nikezić theories

In the domain of higher energies of the hydrogen projectiles, the stopping power exhibits a behavior that is the same in every mentioned theory as they are powerful enough to go through matter, without much interacting with it. Thus, due to the great speed of the projectiles, the interaction with matter electrons is diminished and the influence of angular momentum and spin during the interaction is reduced, so the effects of our Corollary are not so important. But concerning the results for lower energy, the situation is quite different: as the hydrogen projectiles move at significantly lower velocities as they move through the matter than in the previous case, the angular momentum and spin manage to contribute a lot more to the interaction, so the Corollary can be applied, thus explaining why the Stevanović-Nikezić theory [7] is closer to the data obtained from the SRIM then from other theories.

CONCLUSIONS

The corollary of the Noether's theorem, proven in [2] and briefly presented here, enables us to establish the criteria for using mixed theories. It is more qualitative than quantitative criteria, but it can be a good guide when dealing in the area of mixed theories, which are very often used in the field of atomic, molecular and optical physics and radiation physics, also.

Here we have shown how the degree, to which the theory has been mixed, influences its results. The best fit with the experimental results exhibits the Stevanović-Nikezić's theory which is fully quantized (and so not mixed at all), then the Carera-Trujillo's approach that is slightly mixed (because it treats the projectiles as partially stripped classical ions), and finally the Bethe's theory, which is the most extreme example of the mixed theories amongst all three, and the one that shows the most discrepancy with the SRIM data.

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AUTHOR CONTRIBUTIONS

The idea for obtaining the corollary came from V. M. Ristić, who together with M. M. Radulović carried out the proof of the corollary, while T. B. Miladinović, and J. M. Stevanović explored the possibility of applying the corollary. All authors analyzed and discussed the results. The manuscript was written jointly by all authors.

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СТИЦАЊЕ ДУБЉЕГ УВИДА У ПРОБЛЕМЕ ЗАУСТАВНЕ МОЋИ У РАДИЈАЦИОНОЈ ФИЗИЦИ КОРИШЋЕЊЕМ КОРОЛАРА НЕТЕРИНЕ ТЕОРЕМЕ

Теорије које комбинују два различита приступа у разматрању интерагујућих тела, на пример, ако третирају електромагнетно поље ласера класично, а интерагујући атом као квантни објекат, поседују одређене недоследности, и као такве би требало да се називају "мешовитим". Из королара Нетерине теореме (који смо доказали раније) о законима одржања енергије, импулса и момента импулса у мешовитим теоријама, следи да поменуте теорије не подржавају закон одржања момента импулса/спина (прецизније, добијен резултат не значи да закон одржања момента импулса и спина не важи као такав, већ да мешовите теорије могу дати резултате који би могли да наруше поменути закон). У овом раду дато је додатно објашњење, које следи из нашег королара, зашто израчунавање зауставне моћи у потпуно квантној теорији даје боље резултате од оних који се добијају у мешовитим теоријама, што додатно потврђује предвиђања нашег королара.

Кључне речи: зауставна моћ, квантни хармонијски осцилатор, Нетерина теорема, мешовита теорија, закон одржања