

PHOTON ALBEDO COEFFICIENTS AS FUNCTIONS OF μ/Z_{eff} PARAMETER

by

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This paper presents the results of the analyses of photon reflection from planar targets for normal photon incidence and for different shielding materials (water, concrete, aluminum, iron, and copper), in the range of the initial photon energies from 20 keV to 300 keV. Calculations of photon reflection parameters based on the results of Monte Carlo simulations of the photon transport have been performed using MCNP4C code. Integral reflection coefficients, presented as functions of the ratio of total cross-section of photons and effective atomic number of target material, show universal behaviour for all the analysed shielding materials in the selected energy domain.

Key words: photon, total number albedo, Monte Carlo simulation, MCNP code

INTRODUCTION

Integral number and energy reflection coefficients for different particles (ions, electrons, neutrons, and photons) can be presented as universal functions within specified ranges of the initial energies of incident particles. As a consequence, a single function can successfully describe reflection of one type of particles from different target materials within the defined energy range. Number of papers has been published on that topic in the scientific literature over a longer period of time [1-8]. In this paper, reflection of photons from typical shielding materials (water, concrete, aluminum, iron, and copper) for initial photon energies ranging from 20 keV to 300 keV, is considered.

In the previously published papers [5, 6] we determined the number and energy albedo coefficients for several materials of importance for specific applications of photon radiation where albedo problems are of interest. Results presented there, as well as in this paper, cover the interval of initial photon energies E_0 from 20 keV up to 300 keV. Calculated albedo parameters are valid for normal incident beam of photons which targets a thick homogeneous slab of a shielding material. The beam is then partly reflected from the

material while some of the photons are absorbed by the targeted material.

The photon integral reflection coefficients (albedo coefficients) are determined on the basis of Monte Carlo code MCNP4C transport simulation [9]. The MCNP4C also provides detailed angular and energy distributions of reflected photons, with separation of the contributions to the total reflection coming from photons undergoing different number of collisions in the material.

This “collision by collision” analysis enables determination of the mean number of photon collisions prior to the final photon reflection, for different materials and for different initial photon energies. The mean number of photon collisions appeared to be a suitable parameter for the analyses of the universal characteristics of the reflection phenomena [4, 7].

In this paper analyses are done using the parameter μ/Z_{eff} , (where μ is the total cross-section of photons and Z_{eff} the effective atomic number of target material), *i. e.* the albedo coefficients and the mean number of collisions are presented as functions of the μ/Z_{eff} parameter. We confirmed the universal behaviour of the integral reflection coefficients when they are presented as function of μ/Z_{eff} , which brings all the individual curves for different materials close to one universal curve. Analytical expressions for the universal curves are proposed.

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PHOTON REFLECTION AS A FUNCTION OF μ/Z_{eff} PARAMETER

In our previously published paper [7] it was shown that it was possible to present the number and energy albedo of photons as functions of the mean number of photon collisions in the material and that these functions were universal for several shielding materials. The mean number of photon collisions, however, is not known beforehand for a given material and initial photon energy and has to be determined by detailed and time consuming Monte Carlo simulations “collision by collision” for each shielding material and for each initial photon energy, individually.

In order to eliminate the need of knowing the mean number of collisions in the analyses of the photon reflection, and thus the need for Monte Carlo simulations, we made an effort to establish relations between the integral reflection coefficients and some basic parameters describing the photon transport in the target material, which are always well known in advance. Such parameters are characteristics of the incident photon beam and the target material: initial photon energy E_0 , effective atomic number of the target material Z_{eff} and the total cross-section for photon interaction in the target material $\mu = \mu(E_0)$.

Values of Z_{eff} for different materials analysed are given in tab. 1, where Z_{eff} for compounds (water) and mixtures (concrete) are determined according to

$$Z_{\text{eff}} = \left(\sum_{i=1}^n \varphi_i Z_i^{2.94} \right)^{1/2.94} \quad (1)$$

Here, φ_i is the fraction of the total number of electrons which belongs to the element i , and Z_i is the atomic number of the element i . Such a way of calculating energy independent Z_{eff} values has been used for the applications within the energy range of X-rays, but also gives good results in our analyses in the domain up to 300 keV. More robust model for calculating Z_{eff} is presented in ref. [10].

Table 1. Effective atomic number Z_{eff} for the main shielding materials analysed

Material	Z_{eff}
Water	7.42
Concrete	12.55
Aluminum	13
Iron	26
Copper	29

Figure 1 presents graphically the dependence of the parameter μ/Z_{eff} on the initial photon energy E_0 for five different target materials. The energy dependent values for μ are taken from ref. [11].

In preliminary analyses several parameters were proposed to, relatively successfully, present the integral reflection coefficients for different shielding ma-

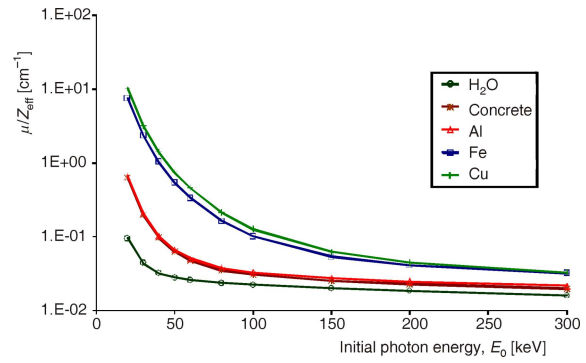


Figure 1. Ratio of the total cross-section for photon interaction μ and the effective atomic number Z_{eff} as a function of the initial photon energy E_0

terials in a universal way. It was determined that the parameter μ/Z_{eff} , defined as a ratio of the total cross-section for photon interaction μ and the effective atomic number of the target material Z_{eff} , provides for the best grouping of all the graphs for different target materials into one universal curve, valid for the analysed photon energy domain.

Figure 2 shows the dependence of the mean number of photon collisions \bar{n} on the parameter μ/Z_{eff} , for all the materials considered. The graph shows also a suitable fitting function, presented as a sum of exponential terms. Table 2 gives absolute and relative discrepancies of the values determined based on the Monte Carlo simulations from the corresponding values of the universal fit function.

The proposed fit function for determination of the mean number of photon collisions \bar{n} as a function of the parameter μ/Z_{eff} is

$$\bar{n} = 1.04773 \cdot 9.37232 e^{\frac{\mu/Z_{\text{eff}} - 0.00383}{0.0101}} + 0.55995 e^{\frac{\mu/Z_{\text{eff}} - 0.00383}{0.11069}} \quad (2)$$

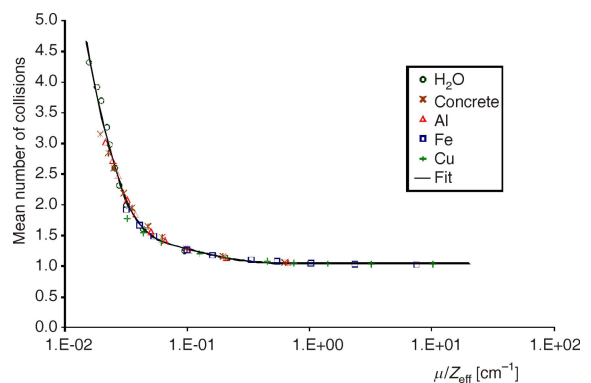


Figure 2. Mean number of photon collisions in the material before the final reflection as a function of the parameter μ/Z_{eff}

Table 2. Absolute and relative discrepancies between the Monte Carlo and “fit” values for the mean number of collisions

Material	E_0 [keV]	\bar{n} Fit – MC*	(Fit – MC) 100/MC [%]
Water	20	0.053	4.311
	30	0.017	1.066
	40	0.064	3.201
	50	0.047	2.017
	60	-0.029	-1.101
	80	-0.140	-4.702
	100	-0.221	-6.781
	150	-0.265	-7.174
	200	-0.151	-3.842
	300	0.066	1.524
Concrete	20	-0.013	-1.254
	30	-0.017	-1.450
	40	-0.016	-1.215
	50	-0.068	-4.617
	60	-0.096	-5.857
	80	-0.063	-3.248
	100	-0.035	-1.599
	150	0.029	1.098
	200	0.119	4.179
	300	0.354	11.259
Aluminum	20	-0.007	-0.682
	30	-0.003	-0.246
	40	0.016	1.257
	50	-0.025	-1.754
	60	-0.057	-3.641
	80	-0.060	-3.257
	100	-0.059	-2.842
	150	-0.077	-3.098
	200	-0.027	-0.986
	300	0.148	4.879
Iron	20	0.024	2.377
	30	0.019	1.813
	40	0.002	0.188
	50	-0.017	-1.624
	60	-0.026	-2.360
	80	0.005	0.431
	100	0.024	1.892
	150	-0.012	-0.829
	200	0.021	1.281
	300	0.135	7.040
Copper	20	0.020	1.907
	30	0.013	1.242
	40	0.004	0.353
	50	-0.008	-0.721
	60	-0.022	-2.011
	80	-0.004	-0.373
	100	0.028	2.304
	150	0.024	1.768
	200	0.064	4.108
	300	0.240	13.485

*MC means the Monte Carlo simulation results for \bar{n}

For all the materials but water maximal discrepancies exist at the upper limit of the considered energy range ($E_0 = 300$ keV) and they range from 4.9% for

aluminum to 13.5% for copper. Most of the values determined based on the fit function for all the materials are within 5% of the Monte Carlo values. This enables reliable reconstruction of the main number of collisions in the energy range up to 200 keV, which can then be applied for determination of the total number albedo, using analytical formulas proposed in [7].

Further analyses showed that the total number albedo a_N in the domain of initial photon energies up to 300 keV can successfully be determined directly from the parameter μ/Z_{eff} , without a need to perform two-step reconstruction: $\mu/Z_{\text{eff}} \rightarrow \bar{n} \rightarrow a_N$. Figure 3 shows the total number albedo of photons for different materials, presented as a function of the parameter μ/Z_{eff} . Good confluence of all the points for different materials is achieved, so the universal analytical formula can be found by adequate fitting.

Analytical expression for the total number albedo of photons a_N , determined by fitting the results of the numerical simulations to a sum of exponential terms, is

$$a_N = 0.09951 \cdot 0.94767 e^{\frac{\mu/Z_{\text{eff}} - 0.00057}{0.01721}} + 0.05623 e^{\frac{\mu/Z_{\text{eff}} - 0.00057}{0.22035}} + 0.093695 e^{\frac{\mu/Z_{\text{eff}} - 0.00057}{20.30851}} \quad (3)$$

From fig. 3 it is obvious that the points do not lie along a common curve so well as in the case of the mean number of photon collision \bar{n} , so the discrepancies of the Monte Carlo results and the ones the universal curve gives are higher. Table 3 presents absolute and relative discrepancies of the values determined based on the Monte Carlo simulations from the corresponding values of the universal fit function.

The presented values of the absolute and relative discrepancies illustrate that the total number albedo, when presented as a function of μ/Z_{eff} is not satisfactorily reproduced over the entire energy domain analysed, especially for heavier materials. There are higher discrepancies close to the borders of the energy interval 20 keV-300 keV. For the energies close to the

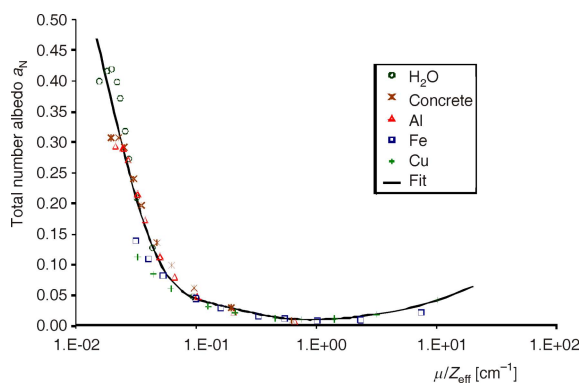


Figure 3. Total number albedo of photon as a function of μ/Z_{eff}

Table 3. Absolute and relative discrepancies between the Monte Carlo and “fit” values for the total number albedo a_N

Material	E_0 [keV]	a_N Fit – MC*	(Fit – MC) 100/MC [%]
Water	20	-0.001	-2.275
	30	0.002	1.248
	40	-0.003	-1.213
	50	-0.023	-8.543
	60	-0.044	-13.895
	80	-0.066	-17.774
	100	-0.072	-18.052
	150	-0.053	-12.736
	200	-0.020	-4.716
	300	0.048	12.037
Concrete	20	0.002	18.760
	30	-0.001	-2.927
	40	-0.016	-26.016
	50	-0.024	-24.736
	60	-0.022	-16.474
	80	-0.017	-8.536
	100	-0.018	-7.430
	150	-0.010	-3.266
	200	0.011	3.614
	300	0.063	20.608
Aluminum	20	0.004	59.267
	30	0.006	23.681
	40	-0.005	-9.632
	50	-0.012	-14.985
	60	-0.011	-9.726
	80	-0.010	-5.498
	100	-0.014	-6.576
	150	-0.016	-6.028
	200	-0.001	-0.304
	300	0.046	15.553
Iron	20	0.012	55.660
	30	0.006	57.827
	40	0.002	23.230
	50	0.002	13.231
	60	0.003	21.517
	80	0.005	15.652
	100	0.001	1.638
	150	0.012	15.246
	200	0.035	32.265
	300	0.069	49.993
Copper	20	0.000	-0.597
	30	0.001	4.692
	40	0.000	4.097
	50	0.000	-0.349
	60	0.002	13.230
	80	0.007	31.090
	100	0.006	19.462
	150	0.014	23.429
	200	0.043	50.094
	300	0.089	79.193

*MC means the Monte Carlo simulation results for a_N

lower limit of the energy domain (20 keV) the absolute values of the total number albedo of photons are low, so that a deduction of two small values results in

higher relative discrepancies. For the higher energies the proposed analytical formula does not reconstruct the “saturation” of the a_N which occurs for water above 100 keV and for concrete above 150 keV (fig. 1 in ref. [7]). The fitting curve anticipates a continuous increase of the total number albedo with the increase of E_0 , which corresponds to decrease of the parameter μ/Z_{eff} . This is not the case for water and concrete above 150 keV, but still the discrepancies for these two light materials are within 20% and 25%, respectively, over the whole energy domain.

In case of metals (aluminum, iron, and copper), discrepancies above 20% exist only in a couple of points close to the boundaries of the energy domain analysed. If these points are discarded, then the values predicted by the fit are well within 20% from the Monte Carlo results for all the materials and for all the remaining initial photon energies.

Known discrepancies between the Monte Carlo based results and the results reconstructed from the universal analytical function give a possibility to determine the energy domains for which the proposed universal representation is satisfactory and acceptable for practical applications.

NUMBER ALBEDO WITHOUT CONTRIBUTION OF THE ONCE SCATTERED PHOTONS

In the domain of initial photon energies up to 300 keV, the dominant contribution to the total photon reflection is coming from the photons backscattered one time. For heavy targets as iron and copper, this component of the total reflection is the major one in terms of absolute values (>50% of the total reflection), while for light materials as water and concrete, it is the major one in relative terms (higher than the contribution from photons scattered twice, or three times, etc.). It is well known that the flux of particles reflected from the material after one scattering can be determined exactly by analytical means. That led us to the idea to subtract this “analytical” component from the total reflected photon flux, and to try to determine the integral reflection coefficients more precisely by fitting only the numerical results which correspond to the total reflection component coming from particles scattered more than once in the target material. Here we describe two options for presenting the photon number albedo: (a) using the difference of the total reflection and the reflection after only one scattering, and (b) using the ratio of these two components of the reflected photons.

Figure 4 presents the number albedo of photons scattered at least twice, $a_{N_{\text{tot}}} - a_{N_1}$, as a function of the parameter μ/Z_{eff} , where $a_{N_{\text{tot}}}$ denotes the total number albedo (sum of all the photons reflected after any number of scatterings in the target material), and a_{N_1} is the number albedo of photons reflected after only one scattering.

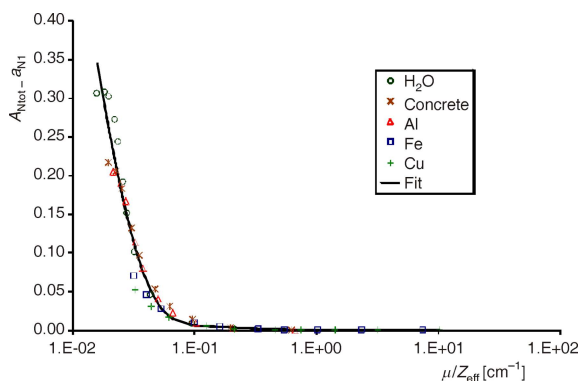


Figure 4. Number albedo of photons scattered at least twice

After fitting the numerical values, obtained by the Monte Carlo simulations, to a sum of exponential terms, $a_{N_{\text{tot}}} - a_{N_1}$ can analytically be presented as

$$a_{N_{\text{tot}}} - a_{N_1} = 0.00045 \cdot 2.35326 e^{-\frac{\mu/Z_{\text{eff}} \cdot 0.01081}{0.01372}} + 0.00904 e^{-\frac{\mu/Z_{\text{eff}} \cdot 0.01081}{0.22414}} \quad (4)$$

Figure 5 provides an alternative representation, option (b), using the ratio $a_{N_1}/a_{N_{\text{tot}}}$, which corresponds to the fraction of number albedo coming from photons scattered only once.

It is evident from figs. 4 and 5 that the quality of the fit is better when the ratio $a_{N_1}/a_{N_{\text{tot}}}$ is used case (b), so the analysis of the absolute and relative discrepancies, presented in tab. 4, is given for the case (b) only.

Figure 5 illustrates that the fraction of the number albedo, which comes from the once scattered photons, is absolutely dominant for low initial photon energies ($\mu/Z_{\text{eff}} > 0.1$) and for heavy targets (iron and copper), while only for lighter materials (water, concrete, and aluminum) and for high initial photon energies its contribution to the total reflection falls below 30%.

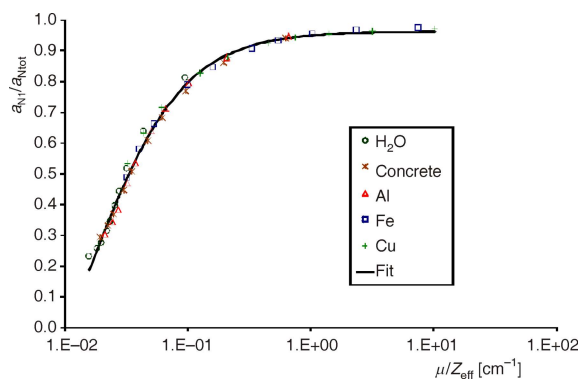


Figure 5. Fraction of the number albedo $a_{N_1}/a_{N_{\text{tot}}}$ as a function of μ/Z_{eff}

Table 4. Absolute and relative discrepancies between the Monte Carlo and “fit” values for the ratio $a_N/a_{N_{\text{tot}}}$

Material	E_0 [keV]	$a_N/a_{N_{\text{tot}}}$ Fit - MC*	(Fit - MC) 100/MC [%]
Water	20	0.006	1.049
	30	0.010	2.035
	40	-0.043	-8.058
	50	0.019	4.110
	60	-0.033	-6.374
	80	-0.004	-0.913
	100	0.017	3.723
	150	-0.014	-3.212
	200	0.036	9.432
300	0.002	0.446	
Concrete	20	0.007	0.734
	30	0.012	1.326
	40	0.010	1.134
	50	0.007	0.815
	60	0.019	2.249
	80	0.016	1.836
	100	0.003	0.372
	150	0.005	0.590
	200	0.006	0.778
300	0.021	2.761	
Aluminum	20	-0.024	-2.952
	30	-0.002	-0.231
	40	0.014	2.098
	50	-0.022	-3.106
	60	-0.009	-1.338
	80	-0.003	-0.533
	100	-0.010	1.672
	150	-0.036	-5.700
	200	-0.044	-6.906
300	0.010	-1.745	
Iron	20	-0.009	-0.926
	30	-0.013	-1.356
	40	-0.004	-0.401
	50	-0.010	-1.007
	60	-0.002	-0.177
	80	-0.004	-0.422
	100	0.002	0.166
	150	-0.004	-0.454
	200	0.000	-0.015
300	0.005	0.486	
Copper	20	0.020	5.339
	30	0.034	9.914
	40	0.016	4.527
	50	0.012	3.661
	60	0.020	6.344
	80	0.015	4.827
	100	0.011	4.131
	150	-0.013	-4.394
	200	-0.006	-2.416
300	-0.044	-18.924	

*MC means the result for $a_N/a_{N_{\text{tot}}}$ calculated from Monte Carlo simulations

Analytical function proposed here for fitting the numerical values of the fraction $a_{N_1}/a_{N_{\text{tot}}}$ is

$$\frac{a_{\text{N1}}}{a_{\text{Ntot}}} = \frac{A_1 A_2}{1 + \frac{\mu/Z_{\text{eff}}}{x_0}^p} A_2 \quad (5)$$

where the coefficients have the following values: $A_1 = -0.57405$, $A_2 = 0.96296$, $x_0 = 0.01613$, $p = 1.15364$.

There are not too many points for all the materials and all the initial photon energies where the relative discrepancies are higher than 5% (3 for water, 2 for aluminum and 4 for copper) and only one point with discrepancy above 10% (copper for 300 keV). Speaking in the μ/Z_{eff} domain, discarding couple of points at the domain boundaries, the relative discrepancies between the numerical values and the analytical function (5) are acceptable: for the region $\mu/Z_{\text{eff}} < 0.1$ they are below 6%, and for $\mu/Z_{\text{eff}} > 0.1$ below 1%.

CONCLUSIONS

Universal characteristics of the integral photon reflection coefficients for normal photon incidence have been analysed in this paper and the integral reflection coefficients are presented as function of the parameter μ/Z_{eff} . This parameter has been proposed as it contains the quantities describing the characteristics of the target material (μ , Z_{eff}) and the incident photon beam (μ as a function of E_0). The parameter μ/Z_{eff} is always known for any combination of the target material and the incident photon energy, so it does not require complex numerical calculations to be determined as it was the case when the main number of photon collisions was used as a parameter to describe the photon reflection coefficients. Five shielding materials were analysed in the domain of the incident photon energies from 20 keV to 300 keV. By using such a parameter, we demonstrated the universal behaviour of several integral photon albedo coefficients, which yields a possibility of establishing analytical models for approximative determination of these reflection coefficients eliminating thus the need for complex Monte Carlo simulations of the photon transport. The quality of such models and its applicability for different materials and initial photon energies was discussed based on the absolute and relative discrepancies of the analytical results from the referent Monte Carlo results.

AUTHOR CONTRIBUTIONS

The selection of materials and the domain of initial photon energies for analyses was done by R. D. Simović. Monte Carlo simulations were prepared, carried out and post-processed by V. L. Ljubenov. The fitting parameter was proposed by V. L. Ljubenov, who performed the fitting as well. Both authors analysed and discussed the results. The manuscript was prepared jointly by both authors. The figures and tables were prepared by V. L. Ljubenov.

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АЛБЕДО КОЕФИЦИЈЕНТИ ФОТОНА КАО ФУНКЦИЈЕ μ/Z_{eff} ПАРАМЕТРА

У раду су изложени резултати анализа рефлексије фотона од равне мете при упаду фотона под правим углом, за различите заштитне материјале (воду, бетон, алуминијум, гвожђе и бакар) и у опсегу почетних енергија фотона од 20 keV до 300 keV. Прорачуни рефлексије заснивају се на Монте Карло симулацијама транспорта фотона које су обављене програмом MCNP4C. Израчунати интегрални коефицијенти рефлексије, приказани у зависности од погодне изабраног параметра μ/Z_{eff} , универзалне су функције за типичне заштитне материјале у одабраној енергетској области.

Кључне речи: фотон, укупни чесиични алbedo, Монте Карло симулација, MCNP програм
