INFLUENCE OF THE PLAIN-PARALLEL ELECTRODE SURFACE DIMENSIONS ON THE TYPE A MEASUREMENT UNCERTAINTY OF GM COUNTER

by

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This paper investigates, through theory and experiment, the influence of the plain-parallel electrode surface dimensions change on the type A measurement uncertainty of a GM counter. The possibilities of applying these results to practical structures are examined by using the methods of mathematical statistics. Special attention is devoted to the influence of electrode surface enlargement on the statistical behavior of the pulse number random variable, expressed in the form of the enlargement law. In the theoretical part of the paper, the general surface enlargement law is derived. Comparison of experimental results with those predicted by the surface enlargement law proved its validity for expressing the type A measurement uncertainty of GM counters constructed with a plain-parallel electrode configuration with a homogenous electric field.

Key words: GM counter, measurement uncertainty, enlargement law, plain-parallel electrodes

INTRODUCTION

The influence of the tube volume on the type A measurement uncertainty of a GM counter has been theoretically investigated within the previous paper [1]. The expressions for dependence of the type A measurement uncertainty on the surface and volume size of the GM counter's tube have been derived within the mentioned paper. Those derived expressions are based on the probability enlargement law [2-4], but without considering the tube geometry, *i. e.* the shape of electrical field within the tube. Since the electrode configuration within the tube is made either with plain-parallel electrodes (homogeneous electrical field) or coaxial electrodes (inhomogeneous electrical field), these two different cases need to be separately considered [5-7], because the previously derived general case, applied both on homogeneous and inhomogeneous electrical field, does not give necessarily the same result.

The aim of this paper is to derive the expression which describes the influence of the plain-parallel electrode surface dimensions change on the type A measurement uncertainty of a GM counter as well as to verify the obtained expression experimentally.

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DEPENDANCE OF THE TYPE A MEASUREMENT UNCERTAINTY ON THE PLAIN-PARALLEL ELECTRODE SURFACE DIMENSIONS

In essence the GM counter's tube is the two-electrode structure with recovering insulator which works on the principle of structure breakdown [8-10]. From the statistical point of view the problem of the three dimensional size change (enlargement or reduction) or the lasting change of an observed occurrence can be solved by applying the Enlargement law. Enlargement law represents the practical application of the multiplication law valid for independent probabilities, by assuming the non-dependence of discharge processes, which take place in parallel with respect to space and time, or consecutively when the time is extended.

When observing an one plain-parallel system with breakdown probability p_1 then the probability of breakdown in the n times enlarged insulating structure p_n is derived directly from the multiplication law valid for independent probabilities, by considering complementary non-breakdown events [2-4]. Non-breakdown of the n times enlarged insulation structure presupposes non-breakdown of all the basic insulation components $(1-p_1)_i$, $i = 1, ..., n$, so

$$p_n = 1 - (1 - p_1)^n \tag{1}$$

Considering probability distribution functions $F_1(x)$ and $F_n(x)$ instead of discrete probabilities, eq. (1) becomes

$$F_n(x) = 1 - [1 - F_1(x)]^n \tag{2}$$

According to eq. (2) the element breakdown probability p_1 joins the distribution function of x used, with parameters α and β, which are initially unknown and must be determined experimentally, but only after the adoption of the type of statistical distribution that corresponds to the observed phenomenon.

Theoretical distributions expected to follow the behavior of the random variable number of pulses of a GM counter are [11-13]:

(1) Normal distribution

$$\Phi(x, \mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{(t-\mu)^2}{2\sigma^2}} \mathrm{d}t \tag{3}$$

with parameters μ (actual, central value = median = = mode) and σ^2 (> 0, variance, σ standard deviation); expectation $EX = \mu$ and variance $D^2X = \sigma^2$.

(2) Weibull distribution

$$F(x) = \begin{cases} 1 - \exp\left[-\left(\dfrac{x - x_0}{\eta}\right)^{\delta}\right], & x \ge x_0 \\ 0, & x < x_0 \end{cases} \tag{4}$$

with parameters: $\eta = x_{63} - x_0$ (63% quantile of associated two-parameter distribution), δ (measure of dispersion, Weibull exponent), and x_0 (initial value); expectation

$$EX = x_0 + \eta\Gamma\left(\frac{1}{\delta} + 1\right) \tag{5}$$

and variance

$$D^2X = \eta^2\left[\Gamma\left(\frac{2}{\delta} + 1\right) - \Gamma^2\left(\frac{1}{\delta} + 1\right)\right] \tag{6}$$

where $\Gamma[(1/\delta) + 1]$ is the gamma function available in tabular form.

For $x_0 = 0$ the two-parameter Weibull distribution is derived from the three-parameter distribution. For $x_0 = 0$ and $\delta = 1$ the exponential distribution is derived and represents the special-case of Weibull distribution. Exponential distribution has only one parameter, usually represented by λ ($\lambda = 1/\eta$) with expectation $EX = \eta = 1/\lambda$ and variance $D^2X = \eta^2 = 1/\lambda^2$.

Expression for the variance of all showed theoretical distributions corresponds to the type A measurement uncertainty, but under conditions of non-dependence of the observed random variable on the all other experimental parameters.

THE EXPERIMENT

In order to verify experimentally the influence of GM counter tube dimensions on the measurement uncertainty of type A an experiment has been conceived within which all other components of the budget uncertainty were fixed, or their influence reduced to zero. To achieve that, a dual multi-layer chamber has been created.

A GM counter was placed in the inner chamber with lead wall thickness of 15 cm so that the tube was placed in its wall, the display was located in the opposing wall of the chamber, while the rest was located within the chamber. The lead plate was placed above the display of the GM counter and it was removed only when reading the number of pulses. The lead screen was placed above the counting tube which was moved by a dependent mechanism serving to open accurately a specific part of the counting tube exposed to radiation. The lead chamber was placed into an iron chamber varnished with 20 μm thick layer of nano-nickel-ferrite in combination with insulating varnish. The lead tunnel was placed from the inside to the outside of the iron chamber and used to lead to the radiation source and its holder. The radiation source holder was of a removable type and could carry two sources at the same time.

The attenuation of the electric and magnetic fields was experimentally determined and was over 100 dB in the range from 50 Hz to 1 GHz, so the measurement uncertainty due to over-voltages was reduced to zero. Background radiation was measured and the correction was performed but it was proved that it was very small or zero. By this system construction the influence of background radiation and over-voltages on the measurement uncertainty of type A was avoided. A photo of the complete measuring system is shown in fig. 1.

The experimental procedure for testing the influence of the tube size on the type A measurement uncertainty consisted of the following steps:

(1) placement of the lead screen into specific position against the radiation source,
(2) setting the work point in the optimal position,
(3) measuring the pulse number for 5 minutes,
(4) measuring the dead time of the GM counter by the two sources method,

Figure 1. Measuring system used in the experiment

(5) repeating steps 3 and 4 fifty times,
(6) measuring the background radiation for an hour,
(7) setting the lead screen in the next position, and
(8) repeating the procedure.

The measurement results were processed by a statistical calculation consisting of the following steps:

– pulse number correction due to the background radiation and the counter dead time,
– formation of the statistical sample composed of 50 values of the random variable mean value of the pulse number and the appropriate statistical sample composed of 50 values of the random variable counter's dead time for all series of measurements (for all tube sizes used),
– using of Chauvenet's criterion for rejecting spurious measurement results,
– U-test with a 5% significance level was employed to establish whether random variables belonged to the same random variable,
– χ^2 -test and graphical test were applied for testing the random variables on belonging to normal, Weibull, and double-exponential distribution, and
– expressing the type A measurement uncertainty of statistical samples by comparison of the experimentally obtained values with the theoretically expected values according to the Enlargement law.

RESULTS AND DISCUSSION

The best choice for estimating the relative fluctuation of a random variable is to use variation coefficient, since it represents the most accurate measure of the relative fluctuation.

Inserting expressions for normal, two-parameter, and three-parameter Weibull and exponential distribution into eq. (2) the variance obtained changes according to the expressions given in tab. 1.

Parameter values for calculating the dependence of variation coefficients on the enlargement factor in case of three-parameter Weibull distribution are obtained according to point estimate [11, 12]

$$\eta^* \quad \exp \; \bar{y} \quad \frac{c}{\delta^*} \tag{7}$$

where

$$\delta^* \quad \frac{\pi}{\sqrt{6 s_y}} M \tag{8}$$

$$\bar{y} \quad \frac{1}{n} \sum_{i \; 1}^{n} \ln(x_i \quad x_0) \tag{9}$$

M is a correction factor dependent on the size of the sample (given in tabular form [11]) and c is Euler's constant.

For two-parameter Weibull and double-exponential distribution the above dependence can be obtained from eqs. (7), (8), and (9) by inserting parameter values x_0 =0, *i. e.* $x_0 = 0$ and $\delta = 1$, respectively.

For normal distribution the above dependence is obtained according to point estimate

$$\mu^* \quad \bar{x} \quad \frac{1}{n} \sum_{i \; 1}^{n} x_i \quad x_{50}^*$$

and

$$\sigma^* \quad s^2 \quad \frac{1}{n \quad 1} \sum_{i \; 1}^{n} (x_i \quad \bar{x})^2 \quad (x_{50}^* \quad x_{16}^*)^2$$

where x_{50}^* and x_{16}^* are quantiles of the empirical distribution function.

Figure 2 shows the dependence of the variation coefficient on the enlargement factor for different theoretical distribution functions. Comparison with experimentally obtained values shows that the empirically obtained characteristic follows a three-parameter Weibull distribution.

Figure 3 shows the random variable number of pulses plotted on Weibull probability paper for $n = 1$ and $n = 10$. Figure 4 shows the result of U-test for random variable number of pulses in case of $n = 10$.

Table 1. Dependence of variation coefficient on enlargement factor *n*

Distribution	Conversion from $\frac{s}{\bar{x}}_1$ to $\frac{s}{\bar{x}}_n$	Remarks
(1) Weibull distributin (2 parameters), $x_0 = 0$	$\frac{s}{\bar{x}}_n = \frac{s}{\bar{x}}_1$	Variation coefficient is not dependent on enlargement factor
(2) Weibull distribution (3 parameters)	$\frac{s}{\bar{x}}_n = \frac{s}{\bar{x}}_1 \frac{1}{1 \quad \frac{x_0}{\bar{x}}(\sqrt[\delta]{n} \quad 1)}$	Case (1) applies to $x_0 = 0$ x_0, parameters of Weibull distribution
(3) Double-exponential distribution	$\frac{s}{\bar{x}}_n = \frac{s}{\bar{x}}_1 \frac{1}{1 \quad \frac{s}{\bar{x}}_1 \frac{\sqrt{6}}{\pi} \ln n}$	–
(4) Normal distribution	$\frac{s}{\bar{x}}_n = \frac{s}{\bar{x}}_1 \frac{k_{1n}}{1 \quad \frac{s}{\bar{x}}_1 k_{2n}}$	$k_{1n} \quad \frac{1}{2}(\lambda_{\sqrt[n]{0,84}} \quad \lambda_{\sqrt[n]{0,16}}$ $k_{2n} \frac{1}{2}(\lambda_{\sqrt[n]{0,84}} \quad \lambda_{\sqrt[n]{0,16}}$ $\lambda_{\sqrt[n]{0,84}}$ and $\lambda_{\sqrt[n]{0,16}}$ are quantiles of order $\sqrt[n]{0,84}$ and $\sqrt[n]{0,16}$ of an $N(0,1)$

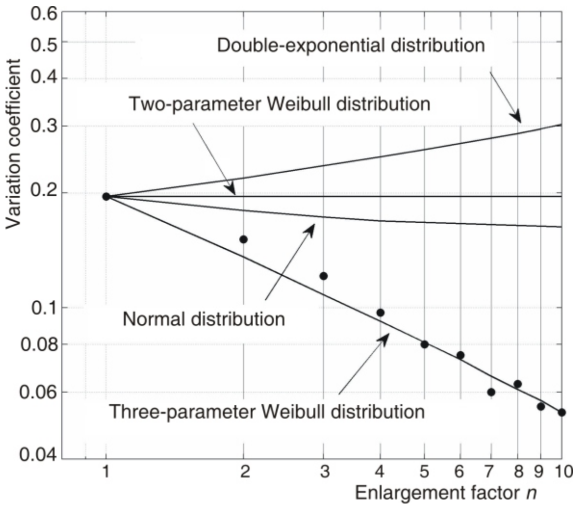

Figure 2. Dependence of variation coefficient on the enlargement factor for different theoretical distribution function; experimentally obtained values are represented with dots

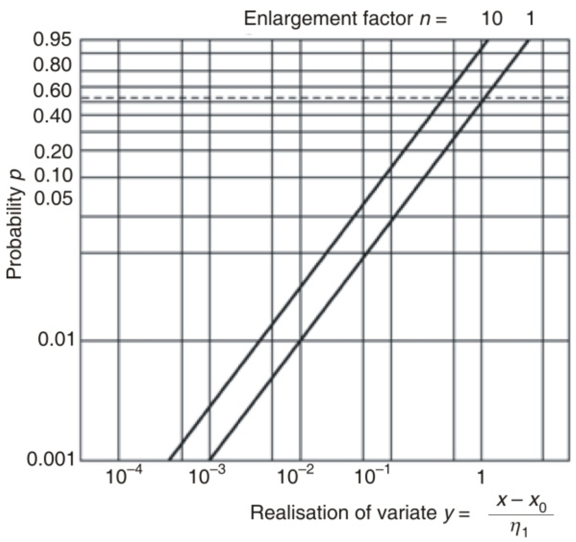

Figure 3. Enlargement law for Weibull distribution, represented in probability paper

The results shown in figs. 3 and 4 verify the result shown in fig. 2, *i. e*. that random variable number of pulses follows the three-parameter Weibull distribution regardless of the electrode surface dimensions.

The Enlargement law is obtained by inserting the expression for three-parameter Weibull distribution into eq. (2)

$$F_n(x) = 1 - \exp\left[-\left(\frac{x - x_0}{\eta_1}\right)^{\delta} n\right] \qquad (10)$$

where η x and δ are the parameters of the initial distribution. From eq. (10) it can be seen that in case of system enlargement for factor *n* there is a change only in the 63% quantile, which is determined by

$$\eta_n = \eta_1 n^{-1/\delta} \qquad (11)$$

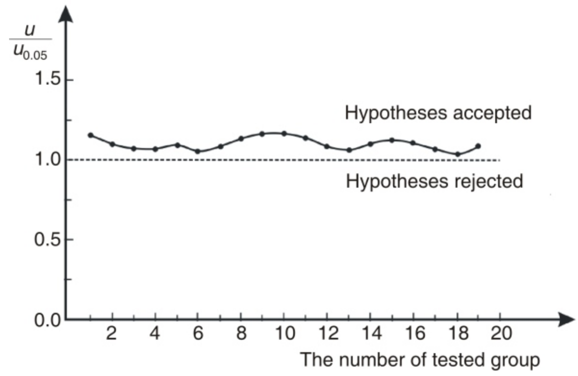

Figure 4. Result of U-test for random variable number of pulses in cese

while the initial value x_0 and Weibull exponent δ remain the same, as shown in fig. 5.

The random variable number of pulses belongs to Weibull distribution, as shown in figs. 2, 3, and 4, so it is possible to determine corresponding expectation and variance dependence, *i. e.* type A measurement uncertainty dependence on the system enlargement factor by using the general expression for Weibull distribution eq. (4). By inserting the eq. (11), which describes the dependence of the 63% quantile on the enlargement factor, into eq. (6) the final expression for standard deviation, scaled to active surface, is

$$\sigma_n = n^{-1/\delta}\sigma_1 \qquad (12)$$

This describes, in the same time, the dependence of type A measurement uncertainty on the GM tube surface dimensions.

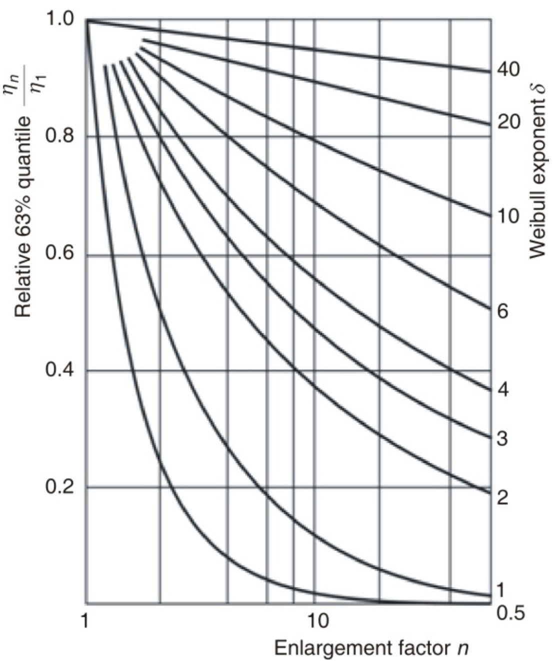

Figure 5. Parameter η_n of Weibull distribution, as a function of the Enlargement law

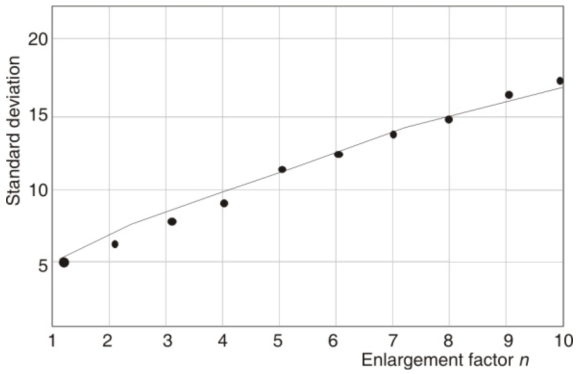

Figure 6. Dependence of the standard deviation of the random variable number of pulses on the enlargement factor *n*

Figure 6 shows the dependence of the standard deviation of the random variable number of pulses on the enlargement factor, obtained by calculating the 63% quantile of Weibull distribution.

CONCLUSIONS

The results presented in this paper demonstrate the behavior of the type A measurement uncertainty in the dependence of counter's tube dimensions with plain-parallel electrodes. The investigation has been done theoretically by applying the probability enlargement law of complementary events. The theoretical algorithm for identification of random variable belonging to theoretical distribution has been developed considering the enlargement law. The obtained expression which describes the behavior of the type A measurement uncertainty has been verified under well controlled laboratory conditions on using a commercial counter's tube, by changing its dimensions with the step of 10%. Extremely good agreement between the theoretical model and the experimental results indicates the validity of the applied theoretical approach. From a practical point of view the findings show that disproportionately reduced type A measurement uncertainty corresponds to GM counters with *n* time reduced counting tube, which allows their reliable operation in the monitoring of punctuated ionizing radiation fields.

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Ковиљка Ђ. СТАНКОВИЋ

УТИЦАЈ ПОВРШИНСКИХ ДИМЕНЗИЈА ЕЛЕКТРОДА ПЛАН-ПАРАЛЕЛНЕ БРОЈАЧКЕ ЦЕВИ ГАЈГЕР-МИЛЕРОВОГ БРОЈАЧА НА МЕРНУ НЕСИГУРНОСТ ТИП А

У раду се, кроз теорију и експеримент, испитује утицај промене површине план-паралелних електрода Гајгер Милеровог бројача на мерну несигурност тип А. Могућност примене ових резултата на практичне структуре испитује се коришћењем метода математичке статистике. Посебна пажња посвећена је утицају повећања површине електрода на статистичко понашање случајне променљиве број импулса, израженом у форми закона пораста. У теоријском делу рада изведен је површински закон пораста у општем облику. Поређење експерименталних резултата са резултатима које предвиђа површински закон пораста показује важење овог закона у изражавању мерне несигурности тип А Гајгер Милеровог бројача израђених у план-паралелној конфигурацији електрода са хомогеним електричним пољем.

Кључне речи: Гајгер Милеров бројач, мерна несигурност, закон пораста, план-паралелне електроде