CONSTRUCTION OF AN AUTOGENERATOR DYNAMIC MODEL APPLICABLE TO NUCLEAR PROCESSES

by

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We propose a new method for constructing a mathematical model of a non-linear system in an auto-oscillation regime. The method is based on the divergence of a vector field having a constant value along the corresponding periodical motion. The variants of the obtained model could be used for describing nuclear processes that are represented by the systems of differential equations analogous to that of the presented model.

Key words: autogenerator, dynamical system modeling, phononic excitations, nuclear processes

INTRODUCTION

A mathematical model of a quasi-periodical motion generator has been proposed in [1], where the model of a linear conservative circuit generator is chosen as a prototype

$$\dot{x} \quad kx, \quad \det k \quad 0$$

For a single degree of freedom, the methods described therein lead to the mathematical model of a harmonic oscillation generator [2]

$$\dot{x} \quad \varpi y \quad \gamma_0 x(G_0 \quad x^2 \quad y^2)$$
$$\dot{x} \quad \varpi x \quad \gamma_0 y(G_0 \quad x^2 \quad y^2)$$

This model is called an Andronov-Vita-Hajkin generator [2]. It should be noted that using a linear conservative circuit as a prototype for autogenerator synthesis limits the possibilities of the proposed approach [1]. The additional drawbacks of this approach stem from the fact that the chosen non-linearities, which are identically equal to zero along the solution being south, cannot always provide the isolation of this solution, or its asymptotic stability. For example, this is true of the following dynamical system

$$\dot{x} \quad y$$
$$\dot{y} \quad x \quad xy(x^2 \quad y^2 \quad 1)^2$$

in which the non-linear term $xy(x^2 + y^2 - 1)$ becomes zero along the solution $x = \cos t$, $y = \sin t$, which belongs to the class of non-isolated periodical motions, both for the first order approximation system, and for the non-linear system constructed on its basis.

In this paper we propose a method for constructing a mathematical model of an autogenerator with one degree of freedom, based on certain characteristics of the divergence of a vector field related to a dynamical system described by two differential equations. The applicability of such a model to nuclear processes is finally proposed.

DIVERGENT CLOSED TRAJECTORIES

Let us discuss a real dynamical system

$$\dot{x} \quad X(x, y)$$
$$\dot{y} \quad Y(x, y) \qquad (1)$$

where functions X and Y are assumed to belong to the class C^k, $k = 1, 2, 3, ...$ in an arbitrary finite region of the R^2 phase plane. A closed trajectory of the dynamical system (1) defined by $\Gamma{:}\gamma(x,y) \quad 0$ is said to be a divergent closed trajectory if the divergency of the vector field determined by the system has a constant value

$$\mathrm{div}Z_{|\gamma \quad 0} \quad \frac{\partial X(x,y)}{\partial x} \quad \frac{\partial Y(x,y)}{\partial y} \quad {}_{\gamma \quad 0} \quad \lambda$$

where $\lambda \quad R$. A divergent closed trajectory can either be an isolated trajectory, *i. e.*, a divergent limit cycle, or be a part of the curves that create a continuum [3-6].

A necessary and sufficient condition for dynamical system (1) to have a simple or complex limit cycle

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is that there exists a non-zero function $B : R^2 \quad R^+$ belonging to class such that the dynamical system

$$\dot{x} \quad X(x, y)B(x, y)$$
$$\dot{y} \quad Y(x, y)B(x, y) \qquad (2)$$

has a simple or complex divergent limit cycle, respectively. This follows from theorems 3 and 4 in [7], with the difference that in [7] functions X and Y are holomorphic, and function B is chosen to belong to class C^{∞}.

A further necessary and sufficient condition for dynamical system (1) to have a simple or complex limit cycle is that there exists a non-zero function $B : R^2 \quad R^+$ belonging to class C^k and a constant $\lambda \quad R \ (R \ )$ different from zero, so that the curve given by

$$\mathrm{div}BZ \quad \frac{\partial[X(x, y)B(x, y)]}{\partial x} \quad \frac{\partial[Y(x, y)B(x, y)]}{\partial y} \qquad (3)$$

has a finite real leg that is a closed trajectory of system (2). This can be proved by the following argumentation. Let be a simple limit cycle of system (1). It follows from the preceding assertion related to equation (2) that there exists a non-zero function $B : R^2 \quad R^+$ belonging to class C^k such that for system (2) the trajectory is a simple divergent cycle. Assuming that the multiplicity of the limit cycle is not changed by passing from system (1) to system (2) or *vice versa*, if B is a positive function, we come to the conclusion that there exists a constant $\lambda \quad R \ (R \ )$ different from zero, such that along equality in eq. (3) is fulfilled. This denotes precisely that the curve described by eq. (3) has a finite real leg that coincides with trajectory , which means that the said condition is indeed a necessary one. The fact that it is also a sufficient condition is demonstrated by the following. Let there exist a non-zero function $B:R^2 \quad R^+$ belonging to class C^k and a constant $\lambda \quad R \ (R \ )$ different from zero, such that the curve of eq. (3) has a finite real leg which is a closed trajectory of system (2). Since function B is positive, curve is also a trajectory of system (1). Let us assume that is not a simple limit cycle, but a trajectory belonging to the class of closed trajectories forming a continuum, or a complex limit cycle of system (1). In that case, however, the properties of the extension function demand that λ equals zero, which contradicts the starting assumption. This means that is indeed a simple limit cycle of system (1).

Let functions X and Y be holomorphic in an arbitrary finite region of the R^2 phase plane, and let for some non-zero function $B : R^2 \quad R$, belonging to class C^{∞}, the curve defined by

$$\frac{\partial[X(x, y)B(x, y)]}{\partial x} \quad \frac{\partial[Y(x, y)B(x, y)]}{\partial y} \quad 0 \qquad (4)$$

has a finite real leg that is a divergent closed trajectory of system (2). If the function at the left-hand side of eq. (4) is of a constant sign in the external or the internal semi-periphery of , then is a complex cycle of system (1). This assertion follows from the Djulak criterion for a double-linked region, the holomorphic properties of functions X and Y, the properties of the extension function, and from the previously stated assertion related to eq. (2).

It should be noted that there are dynamical systems described by equations in the form of (1) with complex limit cycles in internal and/or external semi-periphery for which the vector field divergency changes its sign. An example of such a system is

$$\dot{x} \quad y \quad x \quad y^2 \quad \frac{1}{4} (x^2 \quad y^2 \quad 1)^2$$
$$\dot{y} \quad x$$

with a double divergent limit cycle given by $x^2 + y^2 = 1$. It follows from this that the part of theorem 2 in [7] regarding the necessary condition is false.

DYNAMICAL SYSTEM MODEL

This section provides an example of a dynamical system that serves as a model of harmonic oscillation autogenerators, differing from the generators of Andronov, Vitt and Hajkin. Let

$$\dot{x} \quad a_{10}x \quad a_{01}y \quad a_{30}x^3 \quad a_{21}x^2y \quad a_{12}xy^2 \quad a_{03}y^3$$
$$\dot{y} \quad b_{10}x \quad b_{01}y \quad b_{30}x^3 \quad b_{21}x^2y \quad b_{12}xy^2 \quad b_{03}y^3 \qquad (5)$$

be a non-linear dynamical system with real coefficients $a_{ij}, b_{ij}; i, j = 0, 1, 2, 3$. This system represents a model of sinusoidal oscillation autogenerators (different from those discussed in [6]) if

$$a_{30} \quad a_{10}, a_{21} \quad a_{01}, a_{12} \quad a_{10}, a_{03} \quad a_{01}, a_{01} \quad b_{10}$$
$$b_{30} \quad b_{10}, b_{21} \quad b_{01}, b_{12} \quad b_{10}, b_{03} \quad b_{01} \qquad (6)$$

The proof of this assertion rests upon the Erugin's theorem, according to which the necessary and sufficient condition for system (1) to have an integral curve $\varpi(x, y)$ is that it has the following form

$$\dot{x} \quad F_1(\omega, x, y) \quad \frac{\partial\omega}{\partial y} M(x, y)$$
$$\dot{y} \quad F_2(\omega, x, y) \quad \frac{\partial\omega}{\partial x} M(x, y) \qquad (7)$$

where the functions F_1 and F_2 have the properties $F_1(0, x, y) = 0$, $F_2(0, x, y) = 0$, and $M(x, y)$ is an arbitrary function. Suppose that system (5) generates harmonic (sinusoidal) auto-oscillations. Then it must have an ellipse as its trajectory in the phase plane. Let us assume, for simplicity, that it is a circle given by $x^2 + y^2 = 1$. It follows from Erugin's theorem that the curve given by $\omega \quad x^2 + y^2 - 1 = 0$ is a trajectory of the system

$$\dot{x} \quad 2y(\ a_{01}) \quad (x^2 \quad y^2 \quad 1)(a_{01}y \quad a_{10}x)$$
$$\dot{y} \quad 2x(b_{10}) \quad (x^2 \quad y^2 \quad 1)(b_{10}x \quad b_{01}y) \tag{8}$$

where $a_{01} + b_{10} = 0$, because function $M(x, y)$ is the same in both equations of (7).

Rearranging (8) we obtain

$$\dot{x} \quad a_{10}x \quad b_{10}y \quad a_{10}x^3 \quad b_{10}x^2y \quad a_{10}xy^2 \quad b_{01}y^3$$
$$\dot{y} \quad b_{10}x \quad b_{10}y \quad b_{10}x^3 \quad b_{10}x^2y \quad b_{10}xy^2 \quad b_{01}y^3 \tag{9}$$

Since systems (7) and (9) need to be equivalent, the equality of the corresponding coefficients leads to the following condition

$$a_{30} \quad a_{10}, a_{21} \quad a_{01}, a_{12} \quad a_{10}, a_{03} \quad a_{01}, a_{01} \quad b_{10}$$
$$b_{30} \quad b_{10}, b_{12} \quad b_{01}, b_{12} \quad b_{10}, b_{03} \quad b_{01} \tag{10}$$

These equations do not, however, in general ensure the appearance of auto-oscillations in system (5), because even under the conditions of eq. (10) the curve given by $x^2 + y^2 = 1$ may either belong to the continuum of closed trajectories, or contain the still points of the dynamical system.

If the curve $x^2 + y^2 = 1$ is a trajectory of system (5), then the assertion concerning expression (3) can be used for constructing a model that generates stable oscillations. According to that assertion, the trajectory $x^2 + y^2 = 1$ will correspond to a stable auto-oscillation if a non-zero function $B : R^2 \quad R^+$ can be found, along with a constant $\lambda < 0$, such that divBZ can be analytically presented in the form

$$\frac{\partial[X(x,y)B(x,y)]}{\partial x} \quad \frac{\partial[Y(x,y)B(x,y)]}{\partial y}$$
$$(x^2 \quad y^2 \quad 1)F(x,y) \quad \lambda \tag{11}$$

where F is an arbitrary function.

Considering the aforementioned, we will restrain our attention to the case when function B is given in the form $B(x,y) \quad K(x^2 \quad y^2) \quad \beta, K \quad 0, \beta \quad 0$. Then

$$B(x,y)X(x,y) \quad Ka_{10}x^5 \quad Kb_{10}x^4y \quad 2Ka_{10}x^3y^2$$
$$2Kb_{10}x^2y^3 \quad Ka_{10}xy^4 \quad Kb_{10}y^5 \quad a_{10}(\beta \quad K)x^3$$
$$b_{10}(\beta \quad K)x^2y \quad a_{10}(\beta \quad K)xy^2$$
$$b_{10}(\beta \quad K)y^3 \quad \beta a_{10}x \quad \beta b_{10}y$$

$$B(x,y)Y(x,y) \quad Kb_{10}x^5 \quad Kb_{01}x^4y \quad 2Kb_{10}x^3y^2$$
$$2Kb_{01}x^2y^3 \quad Kb_{10}xy^4 \quad Kb_{01}y^5 \quad b_{10}(\beta \quad K)x^3$$
$$b_{01}(\beta \quad K)x^2y \quad b_{10}(\beta \quad K)xy^2$$
$$b_{01}(\beta \quad K)y^3 \quad \beta b_{10}x \quad \beta b_{01}y$$

and

$$\frac{\partial(BX)}{\partial x} \quad 5Ka_{10}x^4 \quad 4Kb_{10}x^3y \quad 6Ka_{10}x^2y^2$$
$$4Kb_{10}xy^3 \quad Ka_{10}y^4 \quad 3a_{10}(\beta \quad K)x^2$$
$$2b_{10}(\beta \quad K)xy \quad a_{10}(\beta \quad K)y^2 \quad \beta a_{10}$$

$$\frac{\partial(BY)}{\partial y} \quad Kb_{01}x^4 \quad 4Kb_{10}x^3y \quad 6Kb_{01}x^2y^2$$
$$4Kb_{10}xy^3 \quad 5Kb_{01}y^4 \quad b_{01}(\beta \quad K)x^2$$
$$2b_{10}(\beta \quad K)xy \quad 3b_{01}(\beta \quad K)y^2 \quad \beta b_{01}$$

Based on these last expressions and eq. (11), we have

$$\frac{\partial[XB]}{\partial x} \quad \frac{\partial[YB]}{\partial y} \quad K(5a_{10} \quad b_{01})x^4$$
$$6K(a_{10} \quad b_{01})x^2y^2 \quad K(a_{10} \quad 5b_{01})y^4$$
$$(\beta \quad K)(3a_{10} \quad b_{01})x^2 \quad (\beta \quad K)(a_{10} \quad 3b_{01})y^2$$
$$\beta(a_{10} \quad b_{01})$$

and

$$\text{div}BZ \quad (x^2 \quad y^2 \quad 1)[\ K(5a_{10} \quad b_{01})x^2$$
$$K(a_{10} \quad 5b_{01})y^2 \quad (2K \quad 3\beta)a_{10} \quad \beta b_{01}]$$
$$2(K \quad \beta)(b_{01} \quad a_{10})y^2 \quad 2a_{10}(K \quad \beta)$$

In this way, considering relation (11), we come to the conclusion that system (9), or more precisely, system (5), does model the autogenerators of sinusoidal oscillations that are in general different from Andronov-Vitt-Hajkin generators, if

$$2(K \quad \beta)(b_{01} \quad a_{10})y^2 \quad 0$$

i. e., if $a_{10} = b_{01}$. System (9) then becomes

$$\dot{x} \quad a_{10}x \quad b_{10}y \quad a_{10}x^3 \quad b_{10}x^2y \quad a_{10}xy^2 \quad b_{01}y^3$$
$$\dot{y} \quad b_{10}x \quad a_{10}y \quad b_{10}x^3 \quad a_{10}x^2y \quad b_{10}xy^2 \quad a_{10}y^3$$

Following the trend of components miniaturization, the obtained result can be applied for the modeling of photon–phonon interaction within the field of the electromagnetic compatibility of multilayer integrated structures [8, 9].

CONCLUSION

We have demonstrated that the proposed approach can be used in autogenerator synthesis, generating not only harmonic oscillations, but also the periodical oscillations of other arbitrary types. With appropriate modifications, the model could be used for describing interaction of ionizing radiation with phononic excitation modes, as well as for the characterization of various nuclear processes that are represented by the systems of differential equations analogous to that of the presented model.

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КОНСТРУИСАЊЕ ДИНАМИЧКОГ МОДЕЛА АУТОГЕНЕРАТОРА ПРИМЕНЉИВОГ НА НУКЛЕАРНЕ ПРОЦЕСЕ

У раду се износи предлог нове методе за конструисање математичког модела нелинеарног система у ауто-осцилујућем режиму. Метода се заснива на особини дивергенције векторског поља да има константну вредност дуж задатог периодичног кретања. Варијанте овако добијеног модела могу да се користе за описивање нуклеарних процеса који се представљају системима диференцијалних једначина аналогним систему изложеног модела.

Кључне речи: аутогенератор, моделовање динамичким системима, фононске ексцитације, нуклеарни процеси