DETERMINATION OF PLUTONIUM TEMPERATURE USING THE SPECIAL TRANS FUNCTIONS THEORY

by

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Scientific paper
UDC: 546.798.22:517.957
DOI: 10.2298/NTRP1003164P

The problem of estimating plutonium temperature by an iterative procedure based on the special trans functions theory has been studied in some detail. In theory, the differential linear plutonium temperature equation can be effectively reduced to a non-linear functional transcendental equation solvable by special trans functions theory. This approach is practically invariant under the starting plutonium temperature value. This is significant, because the said iterative special trans functions theory does not depend on the password data of the plutonium cargo. Obtained numerical results and graphical simulations confirm the applicability of such approach.

Key words: special trans functions theory, special trans functions, plutonium temperature

INTRODUCTION

With the end of the Cold War, the need for plutonium has decreased. However, the need for developing and upgrading plutonium handling techniques is on the rise. In recent years, the prospect that in the foreseeable future the use and transportation of plutonium irradiated nuclear fuel and radioactive wastes could turn out to be a fact of life has caused intense scrutiny. At present, scientists and engineers are giving their best to come up with as safe as possible modes for the (re)use, transport and (or) stabilization, clean up, and prevention of eventual excesses and dramatic waste pollution incidents. Controlling plutonium temperature is one of the most important issues belonging to the said corpus of scientific and engineering endeavors.

The plutonium temperature differential equation takes the form [1]

$$C\frac{\mathrm{d}T}{\mathrm{d}t} \quad P \quad \frac{T \quad T_c}{R} \qquad (1)$$

or,

$$\tau T'(t) \quad T \quad P_c \quad 0; \quad P_c \quad PR \quad T_c$$

where $\tau = RC$, R being the thermal transport coefficient, C – the being thermo capacity, and P – the energy of the radiation emission, with T_c as thermostat temperature.

On the other hand, we have a very effective iterative special trans functions theory (STFT) approach to the non-linear functional equation of the type

$$Y(t) \quad \tau Y'(t) \quad B(t)\mathrm{e}^{\quad Y(t)} \qquad (2)$$

where $B(t) = P_c\mathrm{e}^{-m\tau}$ that appears in analog forms, [2-4]. Namely, with m being an iterative dissonant time dependent parameter. The meaning of the term dissonant is explained in [2-4].

Let us note that the possible correspondence between eq. (1) and (2) is of utmost importance, since we have an effective iterative procedure for finding the solution to eq. (2).

If we succeed in reducing eq. (1) to eq. (2), the solution to plutonium temperature should be available to us at any time, under the assumption that the initial temperature value/starting condition is unknown. In fact, it is not at all difficult to achieve such a reduction. Namely, after some simple modifications, eq. (2) takes the form

$$\theta_1(t) \quad B(t)\mathrm{e}^{\quad \theta_1(t) \quad \tau Y'(t)} \qquad (3)$$

where

$$\theta_1(t) \quad Y(t) \quad \tau Y'(t) \qquad (4)$$

and, starting eq. (1), after formal modification, by introducing the expression of type

$$K(t) \quad T(t)\mathrm{e}^{\quad K(t)}$$

takes the form

$$\tau K'(t) \quad K(t)[1 \quad \tau K'(t)] \quad P_c\mathrm{e}^{\quad K(t)} \qquad (5)$$

or,

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$$\theta_2(t) \quad \frac{P_c}{1 \quad \tau K'(t)} \mathrm{e}^{\frac{\tau K'(t)}{1 \quad \tau K'(t)}} \mathrm{e}^{\quad \theta_2(t)} \tag{6}$$

where

$$\theta_2(t) \quad k(t) \quad \frac{\tau K'(t)}{1 \quad \tau K'(t)} \tag{7}$$

A formal equalization between eqs. (3) and (6) is possible under the condition that

$$B(t)\mathrm{e}^{\tau Y'(t)} \quad \frac{P_c}{1 \quad \tau K'(t)} \mathrm{e}^{\frac{\tau K'(t)}{1 \quad \tau K'(t)}} \tag{8}$$

which, in turn, can be reduced to the form of

$$B_o(t)\,\mathrm{e}^{\quad Z(t)} \quad 1 \quad Z(t) \tag{9}$$

where

$$B_o(t) \quad \mathrm{e}^{\quad m\tau \quad \tau Y'(t)} \tag{10}$$

and

$$Z(t) \quad \frac{\tau K'(t)}{1 \quad \tau K'(t)} \tag{11}$$

with m as a time-dependent dissonant parameter having to satisfy the error criterion of the form

$$\left|\tau T'(t) \quad T(t) \quad B(t)\mathrm{e}^{m\tau}\right| \quad \varepsilon_m$$

or, more explicitly

$$\left|\tau \quad \frac{T(t \quad \Delta t) \quad T(t)}{\Delta t} \quad T(t) \quad B(t)\mathrm{e}^{m\tau}\right| \quad \varepsilon_m$$

where ε_m is an arbitrary small positive real number.

Consequently, eq. (9) can take the form

$$Z_1(t) \quad B_a \mathrm{e}^{Z_1(t)}; \quad B_a \quad \frac{B_o}{\mathrm{e}^1}; \quad Z_1 \quad 1 \quad Z \tag{12}$$

It is to be pointed out that equation (9) has two real positive solutions under the condition that $B_o < 1$. Thus, from eq. (10) we have $m \quad > Y'(t)$.

Let us note that eq. (5), under above quoted transformations, takes the form

$$Z(t) \quad K(t) \quad P_c[1 \quad Z(t)]\,\mathrm{e}^{\quad K(t)}$$

or the form

$$\begin{aligned} K_1(t) \quad & P_c B_o(t)\mathrm{e}^{-K_1(t)} \\ B(t)\mathrm{e}^{\tau Y'(t)}\mathrm{e}^{\quad K_1(t)} \quad & B_1(t)\,\mathrm{e}^{\quad K_1} \end{aligned} \tag{13}$$

where

$$K_1(t) \quad K(t) \quad Z(t); \; B_1(t) \quad B(t)\mathrm{e}^{\tau Y'(t)} \tag{14}$$

The scheme for the genesis of plutonium temperature from this expression involves eqs. (2), (9), and (13). Namely, we obtain the values of $Y(t)$ and $Y'(t)$ from eq. (2). Then, from eq. (9), we establish $Z(t)$, from eq. (13) we obtain $K_1(t)$ and finally, plutonium temperature $T(t)$. Thus, by the iterative STFT linear differential eq. (1) (the non-linear, multimodal one appears in [16]) has been reduced to the non-linear functional transcendental equation that formally describes processes in non-linear *RC* diode circuits, neutron transport theory, thermionic emission, plasma processes, *etc*. [2-17].

THE ANALYTICAL CLOSED FORM SOLUTION OF EQUATION (13)

This section is organized as follows: first, an overview of the analytical closed form solutions is presented so that the reader may inspect the formulae without having to previously sort out their derivation. A detailed derivation of the formulae is then presented. The outline of the derivation is based on the fact that the STFT approach can be applied for an arbitrary transcendental equation of type (13) (or eq. (12) for $Z_1 > 1$) in a straightforward manner: by determining the suitable partial differential equation for identification with the functional transcendental equation, by finding the analytical closed form solution to the chosen partial differential equation, by predicting the asymptotic solution of the differential equation for identification and, finally, by choosing the optimal equalization between the unique solution and the asymptotic one, [2-17]. The predicted structures of solutions are then examined numerically and by detailed graphical analysis for various parameters of $B_1(t)$ [2, 10, 14].

The transcendental eq. (13), for a given time moment t, takes the simple form

$$\begin{aligned} & K_1(t) \quad B_1(t)\mathrm{e}^{\quad K_1(t)} \\ & K_1(t) \quad 0, \; B_1(t) \quad 0 \end{aligned} \tag{15}$$

The quoted eq. (15) has the solution in the following closed form representation $K_1(t)$ = trans$_+[B_1(t)]$, where trans$_+[B_1(t)]$ is a new special trans function defined as [2-4, 10]

$$\text{trans} \quad [B_1(t)] \quad \lim_{x \quad \infty} B_1 \frac{F \quad (t,x)}{F \quad (t,x \quad 1)} \quad \text{or}$$

$$\text{trans} \quad [B_1(t)] \quad \lim_{x \quad \infty} \ln \frac{F \quad (t,x \quad 1)}{F \quad (t,x)} \tag{16}$$

The practical numerical model for calculating the transcendental number K_1 takes the form

$$\begin{aligned} \langle K_1(t)\rangle_{[P \quad]} \quad & \langle \text{trans} \quad [B_1(t)]\rangle_{[P \quad]} \\ & B_1(t)\frac{F \quad (t,x)}{F \quad (t,x \quad 1)} \quad for\, x \quad x_{\mathrm{g}} \end{aligned} \tag{17}$$

where x_g is the value of x, x [x $(0, \infty)$], when the solution precision defined as $P \quad (x) \quad \log_{10}|G \quad (x)|$ satisfies the inequality $P_+ \quad P_n$, for $x \quad x_g$, the error function is defined as $G_+(x) = K_1 - B_1\,\mathrm{e}^{\quad K_1}$ and satisfies the inequality $|G \quad| \quad g_{\quad m}$ for $x \quad x_g$, while P_m denotes the desired solution precision (desired number of accurate digits), and g_{+m} is an arbitrary small and positive, real number. Consequently, $\langle K_1\rangle_{[P \quad]}$ denotes the numerical value of number K_1 given with $[P_+]$ accurate digits.

The functional series $F_+(t, x)$ takes the form

$$F \quad (t,x) \quad \begin{cases} \displaystyle\sum_{k \quad 0}^{[x]} \frac{B_1^k(t)(x \quad k)^k}{k!} & \text{for } x \quad 0 \\ 0 & \text{for } x \quad 0 \end{cases} \tag{18}$$

where $[x]$ denotes the greatest integer less than or equal to x. Consequently, formula (17) represents the numerical structures for numbers K_1. The choice of x controls the number of accurate digits for constants K_1, and satisfies the error criterion. On the other hand, the number of accurate digits in the numerical structure of constant K_1 is, practically, determined by the physical requirements for the plutonium temperature estimation problem.

THE PARTIAL DIFFERENTIAL EQUATION AS AN EQUATION FOR IDENTIFICATION

Transcendental eq. (15) can be identified with a partial differential equation of the type

$$\frac{\partial F_+(t,x)}{\partial x} = B_1(t)F_+(t,x-1) \tag{19}$$

The partial differential eq. (19) is analytically solvable using a Laplace transform. The unique solution is a series (18). Or, more explicitly, equation (18) takes the form

$$F_+(t,x) = \sum_{k=0}^{\infty} \frac{B_1^k(t)(x-k)^k H(x-k)}{k!} \quad for\, x \geq 0 \tag{20}$$

where $H(x-k)$ is the Heaviside's unit function defined as

$$H(x-k) = \begin{cases} 1 & \text{for } x \geq k \\ 0 & \text{for } x < k \end{cases}$$

On the other hand, the asymptotic solution of the form ([2-4, 10])

$$F_{+p}(t,x) = \mathrm{e}^{\frac{K_1(t)x}{1+K_1(t)}} \tag{21}$$

satisfies the differential eq. (19) under the condition that satisfies the transcendental equation (15).

According to the unique solution principle and function theory, we have

$$\lim_{x\to\infty} \frac{F_+(t,x)}{F_{+p}(t,x)} = 1 \tag{22}$$

AN ANALYSIS OF EQUALITY (22)

Let us note that the essential part of the STFT is the existence of equality (22). Substituting results (18) and (21) for $F_+(t,x)$ and $F_{+p}(t,x)$, respectively, in eq. (22), we obtain

$$\lim_{x\to\infty} \sum_{k=0}^{[x]} \frac{B_1^k (x-k)^k}{k!} \mathrm{e}^{-\frac{K_1 x}{1+K_1}} \neq 0$$

or, since

$$B_1 = K_1 \mathrm{e}^{K_1}$$

we have

$$\lim_{x\to\infty} (1+K_1) \sum_{k}^{[x]} \frac{K_1^k \mathrm{e}^{-K_1(x-k)} (x-k)^k}{k!} = 1$$

With regards to

$$\mathrm{e}^{-K_1(x-k)} = \sum_{m=0}^{\infty} \frac{(-1)^m K_1^m (x-k)^m}{m!}$$

the above equation takes the form

$$\lim_{x\to\infty} (1+K_1) \sum_{k=0}^{[x]} K_1^k \frac{(x-k)^k}{k!} \sum_{m=0}^{\infty} (-1)^m K_1^m \frac{(x-k)^m}{m!} = 1 \tag{23}$$

After a series expansion on the left side, the above equation can be written as:

$$(1+K_1) \lim_{x\to\infty} \left[\sum_{m=0}^{\infty} \frac{(-1)^m K_1^m x^m}{m!} + \sum_{m=0}^{\infty} \frac{(-1)^m K_1^{m+1} (x-1)^{m+1}}{m!1!} + \ldots + \frac{(-1)^k \sum_{m=0}^{\infty} (-1)^m Y^{m+K} (x-K)^{m+K}}{(m!K!)} + \ldots \right] \ldots = 1 \tag{24}$$

From the above quoted, expanded form, it is clear that the term of $(-1)^M K_1^M$ takes the following form

$$\frac{x^M}{M!} + \frac{x^{M-1}}{(M-1)!} - \frac{(x-1)^M}{(M-1)!} - \frac{(x-1)^{M-1}}{(M-2)!} + \frac{(x-2)^M}{(M-2)!2!} + \frac{(x-2)^{M-1}}{(M-3)!2!} - \ldots$$
$$\ldots + (-1)^M \frac{(x-M)^M}{M!} + \frac{(x-M)^{M-1}}{(M-1)!}$$

and, finally, on the left side of the eq. (24), we obtain

$$\sum_{M=0}^{[x]} (-1)^M K_1^M \left[\sum_{k=0}^{M} \frac{(-1)^k (x-k)^M}{(M-k)!k!} + \sum_{k=0}^{M} \frac{(-1)^k (x-k)^{M-1}}{(M-1-k)!k!} \right] = 1 \tag{25}$$

$$\text{for x} \to \infty$$

or, more explicitly

$$\lim_{x\to\infty} \sum_{M=1}^{[x]} (-1)^M K_1^M \left[\sum_{k=0}^{M} \frac{(-1)^k (x-k)^{M-1}}{(M-k)!k!} (x+M) \right] = 0 \tag{26}$$

Note that the equality (26) holds if, and only if

$$S_M(x) = \sum_{k=0}^{M} \frac{(-1)^k (x-k)^{M-1}}{(M-k)!k!} = 0$$
$$\text{for } M \geq 1, \; M = 1,2,3,\ldots[x] \tag{27}$$

By simply using mathematical induction, we can show that eq. (27) is satisfied for any positive integer . Consequently, under the assumption (27) we have

$$S_{M+1}(x) = \sum_{k=0}^{M+1} \frac{(-1)^k (x-k)^M}{(M+1-k)!k!} = 0$$

and

$$S_{M+1}(x) = \sum_{k=0}^{M+1} \frac{(-1)^k M(x-k)^{M+1}}{(M+1-k)!k!}$$

Thus, we obtain

$$S_{M+1}(x) - S_M(x) = S_{M+1}(x)$$

$$\sum_{k=0}^{M} \frac{(-1)^k (x-k)^{M-1}}{(M-k)!k!}\left(\frac{x-k}{M+1-k} - 1\right)$$

$$(-1)^{M+1}\frac{(x-M-1)^M}{(M+1)!} = S_{M+1}(x)\frac{x-M-1}{M}$$

and

$$S_{M+1}(x) = S_{M+1}(0) + \int_0^x \frac{M}{(y-M-1)}\mathrm{d}y \quad S_{M+1}(x) = 0$$

This approach, by means of mathematical induction, is a solid proof of the existence of equality (22).

THE GENERAL TRANS SCHEME

The introducing section makes it clear that, when applying the unique solution principle [2-17], we have the following equality

$$F_p(t,x) = \lim_{x\to\infty}[F_+(t,x)] \text{ and}$$

$$K_1(t) = \lim_{x\to\infty} B_1(t)\frac{F_+(t,x)}{F_+(t,x+1)} = \text{trans}_+[B_1(t)] \tag{28}$$

Analogically, the unique solution principle states that

$$\langle F_+(t,x)\rangle_{[P_+]} = \langle F_p(t,x)\rangle_{[P_+]} \text{ for } x \geq x_g \tag{29}$$

where $F_+(t, x)_{[P+]}$ denotes the numerical value of function $F_+(t, x)$ given with $[P_+]$ accurate digits.

Now, from eq. (29), it becomes possible to establish the equality

$$\left\langle\frac{F_+(t,x+1)}{F_+(t,x)}\right\rangle_{[P_+]} = \left\langle\frac{F_p(t,x+1)}{F_p(t,x)}\right\rangle_{[P_+]} = \left\langle e^{K_1(t)}\right\rangle_{[P_+]}$$

and

$$\langle K_1(t)\rangle_{[P_+]} = \ln\frac{F_+(t,x+1)}{F_+(t,x)} \text{ for } x \geq x_g \tag{30}$$

Let us note that, more explicitly, for a fixed time moment, number K_1 takes the form of

$$\langle K_1\rangle_{[P_+]} = B_1 t\frac{\displaystyle\sum_{k=0}^{[x]}\frac{B_1^k(x-k)}{k!}}{\displaystyle\sum_{k=0}^{[x+1]}\frac{B_1^k(x+1-k)^k}{k!}} \text{ for } x \geq x_g \tag{31}$$

From a theoretical point of view, the solution (31) for K_1 can be found with an arbitrary order of accuracy, by taking an appropriate value of x.

CONCERNING THE SOLUTION TO THE NON-LINEAR MULTIMODAL DIFFERENTIAL EQUATION (2)

Let us note that, in previous sections, the problem of plutonium temperature calculation (1) has been reduced to a solvable non-linear differential eq. (2). In this paper, by applying the special trans function theory (Perovich, 1992, 1995, 1996, 1997, 1999, 2004, 2006, 2007, 2009, *etc.*), eq. (2) has been successfully solved. Namely, as a transcendental equation of the form

$${}^{n}Y(t) = \tau\,{}^{(n)}Y'(t) + B(t)e^{{}^{(n)}Y(t)} \tag{32}$$

where ${}^{(n)}Y(t)$ is the solution to the n-th iteration when ${}^{(n)}Y'(t)$ is a known time functional parameter. Under this assumption, the analytical closed form solution to the eq. (2) takes the form

$$\begin{gathered}{}^{(n)}Y(t) = \text{trans}_+[B_n(t)] + \tau\,{}^{(n)}Y'(t)\\ B_n(t) = B(t)e^{\tau\,{}^{(n)}Y'(t)}\end{gathered} \tag{33}$$

where $\text{trans}_+[B_n(t)]$ is a special trans function [2-4, 10] defined as

$$\text{trans}_+ B_n(t) = \lim_{x\to\infty}\ln\frac{\varphi_+[B_n(t),x-1]}{\varphi_+[B_n(t),x]} \tag{34}$$

where

$$\varphi_+[B_n(t),x] = \sum_{k=0}^{[x]}[B_n(t)]^k\frac{(x-k)^k}{k!} \tag{35}$$

From eq. (32), by differentiation, we have

$$\begin{aligned}{}^{(n)}Y'(t+\Delta t) = {}^{(n)}Y'(t) + \big[&B'(t)e^{{}^{(n)}Y(t)} \\ &+ {}^{(n)}Y'(t)B(t)e^{{}^{(n)}Y(t)} - {}^{(n)}Y'(t)\big]\frac{\Delta t}{\tau}\end{aligned} \tag{36}$$

Now, from eqs. (32), (33), and (36) directly follows that

$${}^{(n)}Y(t+\Delta t) = \text{trans}_+(B_n(t+\Delta t)) + \tau\,{}^{(n)}Y'(t+\Delta t) \tag{37}$$

On the other hand, we have

$${}^{(n+1)}Y'(t) = \frac{{}^{(n)}Y(t+\Delta t) - {}^{(n)}Y(t)}{\Delta t} \tag{38}$$

The error function to the n-th iteration is defined as

$${}^{(n)}g = \left|{}^{(n+1)}Y'(t) - {}^{(n)}Y'(t)\right| \leq \varepsilon \tag{39}$$

where ε is an arbitrary small, real, positive number.

If the error criterion is not satisfied, then the iterative procedure continues, with the new value of the first differentiation, ${}^{(n+1)}Y'(t)$, *etc.*

THE ANALYTICAL CLOSED FORM SOLUTION TO EQUATION (12)

Let us note that it is possible to solve eq. (12) by STFT, as it has been presented in section "The analytical closed form solution of eq. (13)", and [2], for $Z_1 <$ 1, and in [2] and [14], for $Z_1 > 1$.

Namely, for $Z_1 < 1$, the transcendental eq. (12) can be identified with a partial differential equation of the type

$$\frac{\partial F_<(x, B_a)}{\partial x} \quad B_a F_<(x \quad 1, B_a) \quad 0 \tag{40}$$

where $F_<(x, B_a)$ is an arbitrary real function, for $x > 0$, and $F_<(x, B_a) = 0$, for $x < 0$. Consequently, by applying STFT, we have

$$Z_1(t) \quad \mathrm{trans}_< [B_a(t)] \tag{41}$$

where $\mathrm{trans}_< B_a(t)$ is an elementary special tran function defined as [2-4, 10]

$$\mathrm{trans}_< [B_a(t)] \quad \lim_{x \quad \infty} B_a \frac{F_<[B_a(t), x]}{F_<[B_a(t), x \quad 1]} \quad \text{or}$$

$$\mathrm{trans}_< [B_a(t)] \quad \lim_{x \quad \infty} \ln \frac{F_<[B_a(t), x]}{F_<[B_a(t), x \quad 1]} \tag{42}$$

wehere

$$F_<[B_a(t), x] \quad \sum_{k \quad 0}^{[x]} (\quad 1)^k \frac{(x \quad k)^k}{k!}$$

On the other hand, the application of the special trans functions theory (Perovich, 1991-2009), results in $Z_1 > 1$, $Z_{1>}$

$$Z_1 \quad \lim_{\substack{u \quad \infty \\ x \quad \infty}} \ln \frac{F_<(x, B_a) F_>(u \quad a, b)}{F_<(x \quad 1, B_a) F_>(u, b)} \quad \mathrm{trans}_>(B_a a) \tag{43}$$

where $\mathrm{trans}_>(B_a, a)$ is a new special tran function, and

$$b \quad \frac{Z}{a} \tag{44}$$

where $a = 2\ln 3$, or, generaly $a \quad 2\ln[(A \quad 1)/(A \quad 1)]$, where $A = 2, 3, 4, 5$, and 235, $Z_< = \mathrm{tran}_<(B_a)$, where $\mathrm{tran}_<(B_a)$ is an elementary special tran function [2, 14] defined in (42)

$$\begin{aligned} F_>(u, b) &\quad bR(u, b)\mathrm{e}^{bu} \\ R_>(u, b)\mathrm{e}^{bu} &\quad [R(u, b)\mathrm{e}^{bu}] \end{aligned} \tag{45}$$

where

$$R(b, u) \quad \sum_{n \quad 0}^{[u/a]} (\quad 1)^n \frac{(b\mathrm{e}^{\quad ab})^n (u \quad na)^n}{n!} \quad \text{for } u \quad 0 \tag{46}$$

and $[u/a]$ denotes the greatest integer less than or equal to u/a.

For practical calculation purposes, when the value of x is a positive integer M, we have the following formula

$$\langle Z_{1>} \rangle_{[P_>]} \quad \left\langle \ln \frac{F_<(M, B_a)}{F_<(M \quad 1, B_a)} \right\rangle_{[P_>]}$$

$$\left\langle \ln \frac{F_>(u \quad a, b)}{F_>(u, b)} \right\rangle_{[P_>]} \tag{47}$$

where $\langle Z_{1>} \rangle_{[P_>]}$ is the value of the transcendental number $Z_{1>}$ given with $[P_>]$ accurate digits. Finally,

$$\langle Z_{1>} \rangle_{[P_>]} \quad \langle \mathrm{trans}_< (B_a) \rangle_{[P_>]} \quad \langle \mathrm{trans}_> (B_a, a) \rangle_{[P_>]}$$

In accordance with the previous definition, we have that error function defined as

$$G \quad Z \quad B_a \mathrm{e}^Z \tag{48}$$

satisfying the inequality

$$P \quad \log_{10}|G| \quad P_m \tag{49}$$

where P_m denotes the desired solution precision (desired number of accurate digits).

PLUTONIUM TEMPERATURE CALCULATION

Previous sections lead us to the conclusion that it is possible to determine plutonium temperature without the value of the starting temperature or any other temperature value. Namely, according to eq. (32), within the STFT iterative procedure, by means of the error criterion defined in (38), we obtain the solution for $Y(t)$ and $Y(t)$. For the obtained value of $Y(t)$ by STFT, from eq. (9), we come up with

$$Z \quad 1 \quad Z_1 \quad 1 \quad \mathrm{trans}_>(B_a, a) \quad 1 \quad \mathrm{trans}_> \left(\frac{B_o}{\mathrm{e}^1}, a \right)$$

where $\mathrm{trans}_>[B_o/\mathrm{e}^{(1)}, a]$, is given by formula (43).

From eq. (15)

$$K_1(t) \quad B_1(t)\mathrm{e}^{\quad K_1(t)}, \; K_1(t) \quad 0, \; B_1(t) \quad 0$$

by STFT application [2, 14], we obtain the solution to $K_1(t)$ in the form

$$K_1(t) \quad \lim_{x \quad \infty} B_1(t) \frac{F_<(t, x)}{F_<(t, x \quad 1)} \quad \mathrm{trans}_<[B_1(t)]$$

Finally, from eq. (14), the value of $K(t)$ follows in the form of

$$K(t) \quad K_1(t) \quad Z(t) \quad \mathrm{trans}_<[B_1(t)] \quad 1 \quad \mathrm{trans}_>(B_a, a)$$

Also, this leads us to the formula for plutonium temperature

$$\begin{aligned} T &\quad \mathrm{trans}_<[B_1(t)] \quad 1 \quad \mathrm{trans}_> \left(\frac{B_o}{\mathrm{e}^1}, a \right) \\ &\quad \mathrm{e}^{\mathrm{trans}_<[B_1(t)]} \quad 1 \quad \mathrm{trans}_> \left(\frac{B_o}{\mathrm{e}^1}, a \right) \end{aligned} \tag{51}$$

In this case, the fact that the calculation of plutonium temperature is practically invariant of the starting (initial) plutonium temperature is of utmost importance.

NUMERICAL RESULTS

Some important results in this presentation are, in fact, the values of $T(t)$. In tab. 1 we have presented some of the results for $T(t)$, when the values of $B(t)$ vary.

Table 1. Calculating values of $T(t)$ for different parameters $B(t)$

$B(t)\ 10^6$	$T(t)$ [°C]	G
1.198243	610.43	1.740E-09*
1.311565	579.41	1.607E-08
1.787568	386.00	6.243E-08
2.009790	303.75	2.962E-07
2.221021	288.35	4.119E-06
2.666739	217.94	1.082E-05
2.888951	193.90	3.838E-05
3.978307	117.81	2.087E-04

*1.740E-09 means $1.740\ 10^{-9}$

The obtained values of $T(t)$, by STFT, for various parameters $B(t)$, presented in tab. 1, are also presented graphically in figs. 1 and 2.

In tab. 2 we have presented, numerically, the fact that error function depends on the number of iterations.

The correlation between the error function and the number of iterations is presented graphically in fig. 3, as well.

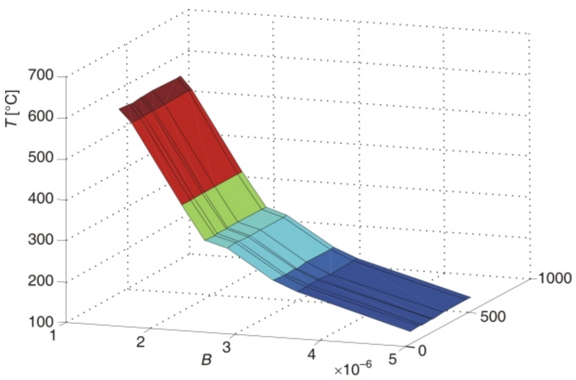

Figure 1. Plutonium temperature being calculated in function of $B(t)$

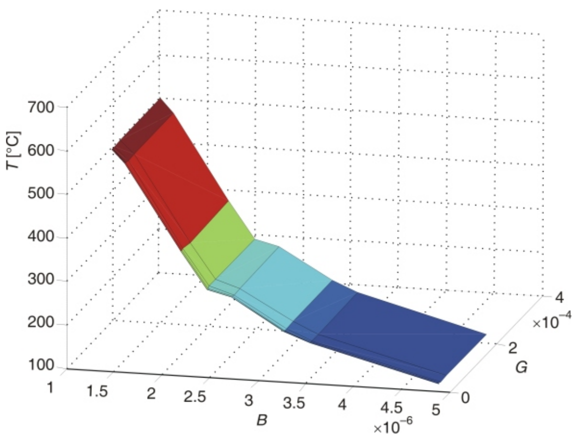

Figure 2. Plutonium temperature being calculated in function of $B(t)$ and G

Table 2. Error function and the corresponding number of iterations

Number of iteration n	Error $^{(n)}g$
1	9.316841032003254 E-6
2	9.005140616302754 E-6
3	8.703880545191112 E-6
4	8.412710164051873 E-6
6	7.859298628787315 E-6
8	7.342338300998580 E-6
10	6.859425865535229 E-6
11	6.630031235133060 E-6
14	5.986925645906638 E-6
15	5.786753312975179 E-6
20	4.882076065193530 E-6
22	4.561204494990534 E-6
25	7.596416564226161 E-6
50	6.408319747741587 E-7
55	4.119083905074206 E-7
100	5.406296559762325 E-8
120	5.225571780620442 E-8
130	5.050898597147580 E-8

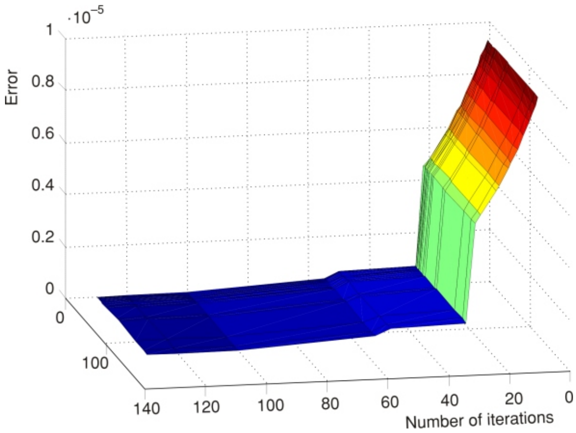

Figure 3. The error function correspondence to the number of STFT iterations

CONCLUSION

It is clear that the calculation of plutonium temperature is of significance since it gives us the possibility of estimating the concentration and mass of the said radioactive element. Plutonium temperature is of great importance for transport safety, as well.

Once again, let us point out that the STFT being presented in this paper and applied for calculating plutonium temperature is independent of password cargo data and the initial plutonium temperature. The use of the STFT procedure also allows us to determine the time changing plutonium temperature with a very high accuracy. In short, it is obvious that the STFT iterative procedure is an excellent method for the said purpose, mostly because it is independent of possible falsified plutonium system data.

Finally, we wish to point out that it is possible to present plutonium temperature in the form of

$$T(t) \quad \psi(t)\mathrm{e}^{\psi(t)t} \tag{52}$$

Accordingly, eq. (2) takes the form

$$\begin{aligned} \tau\psi \ (t) \quad & \tau\psi(t)\psi \ (t)t \\ \psi^{2} \ \ t \quad & P_{c}\mathrm{e}^{\ \psi(t)t} \end{aligned} \tag{53}$$

Now, we can apply an analogous iterative procedure described for eq. (2) to eq. (53). Of course, the equation for identification is more complex, thus the corresponding numerical simulations are, indeed, robust. However, this might be the subject matter of some future theoretical and numerical analysis.

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Received on October 6, 2010
Accepted on October 25, 2010

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ОДРЕЂИВАЊЕ ТЕМПЕРАТУРЕ ПЛУТОНИЈУМА КОРИШЋЕЊЕМ ТЕОРИЈЕ СПЕЦИЈАЛНИХ ТРАНС ФУНКЦИЈА

Проблем одређивања температуре плутонијума, итеративним поступком заснованим на теорији специјалних транс функција, проучен је у неким детаљима. Према овој теорији, линеарна диференцијална једначина за температуру плутонијума може се ефектно редуковати на нелинеарну функционалну трансцендентну једначину, која се решава специјалним транс функцијама. Овај поступак је практично инваријантан од почетне вредности температуре плутонијума. То је веома значајно због тога што наведена итеративна процедура не зависи од битних података за плутонијумски терет. Добијени нумерички резултати и графичке симулације, потврђују примењивост овог приступа заснованог на теорији специјалних транс функција.

Кључне речи: теорија специјалних транс функција, специјалне транс функције, температура плутонијума