EXPANDED AND COMBINED UNCERTAINTY IN MEASUREMENTS BY GM COUNTERS

by

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This paper deals with possible ways of obtaining expanded and combined uncertainty in measurements for four types of GM counters with a same counter's tube, in cases when the contributors of these uncertainties are cosmic background radiation and induced overvoltage phenomena. Nowadays, as a consequence of electromagnetic radiation, the latter phenomenon is especially marked in urban environments. Based on experimental results obtained, it has been established that the uncertainties of an influenced random variable "number of pulses from background radiation" and "number of pulses induced by overvoltage" depend on the technological solution of the counter's reading system and contribute in different ways to the expanded and combined uncertainty in measurements of the applied types of GM counters.

Key words: uncertainty in measurements, GM counter, background radiation, overvoltage phenomenon, dead time

INTRODUCTION

The result of any measurement contains uncertainty, meaning that an ideally true value of a measured variable cannot be known. There are many sources of uncertainties in measurements and all of them cannot be taken into consideration. According to the error theory, the previously predominantly used classical mathematical discipline, information about the error is obtained from the measurement itself. The said value represents the difference between the obtained result and an appropriate value obtained by standard measuring instrumentation. The original variables of the error theory were random and systematic errors (stochastic and deterministic variables), idealized terms no more in practice today [1]. With the introduction of a new concept, that of measurement uncertainty, all errors were defined as stochastic variables and, consequently, a practically oriented method of measuring was created, suitable for efficient application in all sorts of experimental measurements. The distinction between these two concepts can, probably, be best seen in the schematic representation given in fig. 1.

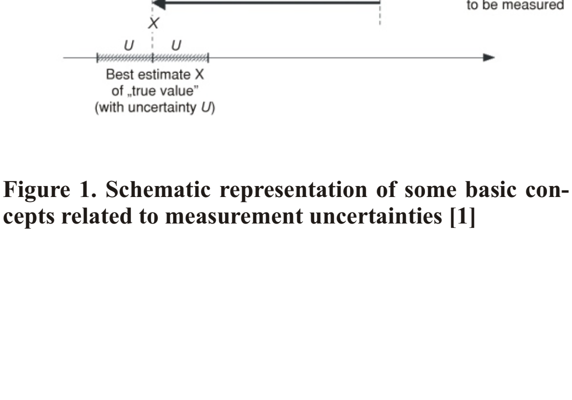

Figure 1. Schematic representation of some basic concepts related to measurement uncertainties [1]

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The expression of measurement uncertainties utilizes the terms and mathematical apparatus of the classical statistical theory. Assigning appropriate distribution and probability functions to each and every datum of uncertainty is the basic assumption of the whole concept. Apart from that, the following terms pertaining to the domain of uncertainty in measurement are also introduced [2]: (1) standard uncertainty, u, which, according to the definition, equals the standard deviation, $u = s$; statistical certainty suitable for standard uncertainty depends on the distribution assigned to a particulate measurement; (2) expanded uncertainty, U, is defined as a product of standard uncertainty and the coverage factor, k, which depends on the assigned distribution, $U = ku$. A high value of statistical certainty, of the order of 99%, is suitable for expanded uncertainty. This means that the measured variable falls within the interval x_s U with high certainty, where x_s is the mean value of the measurement.

According to the method with which the uncertainty is determined, there are two possible types of uncertainties, type A and type B. Type A is determined solely by the statistical method, meaning that it exists in measurements which are performed more then once, in cases when the measuring variable has a stochastical nature. The type B uncertainty is determined by methods other than statistical analysis. This uncertainty type can be determined even for a single measurement, when type A does not exist. It includes uses of all available data and knowledge about measuring equipment, influenced environmental quantities, application of correction factors or physical data taken from literature, *etc*. In this case, it is necessary to assign an appropriate distribution function, since different distributions can be applied. As opposed to type B, in cases of a mean value distribution concerning type A, it is always the Gaussian distribution (according to the Central Limit Theorem), even if a measured stochastic variable belongs to some other distribution. For example, in cases of natural phenomena such as background radiation and radioactive decay of radionuclides, distributions are assigned on the bases of statistical behavior of these phenomena. The Poisson distribution, suitable for low frequency events (appendix A), is applied to background radiation and the Gaussian distribution, suitable for high frequency events (appendix B), is applied to instances of the decay of radionuclides.

The notion of combined uncertainty is introduced in measurements where there is more then one influenced quantity contributing to the said uncertainty. Combined uncertainty is used either in repeated measurements where the uncertainties of type A and type B are simultaneously determined or, in a single measurement (when type A does not exist), where more then one measuring instrument is used and each instrument contributes to the uncertainty of the type B. As a rule, when an experiment is performed, the determination of expanded and combined uncertainty represents the final goal of processing measurement data.

The aim of this paper is to investigate the influence of cosmic background and overvoltage phenomena on measurements performed by different types of GM counters. We believe it is of utmost importance to take into consideration the overvoltage phenomenon in measurements performed in an urban environment (where the electromagnetic radiation is high). The presence of overvoltage during these measurements may well trigger the reading system of the counter and give a higher number of counting pulses than expected. In that sense, when the experiments are performed, the same counter's tube is used, while the technological solution of the electronic devices (counter reading system) is changeable. The applied reading systems were made with the same type of tube technology, discrete or integrated, with or without anticoincidence protection. Appropriate statistical distributions were assigned to the relevant random variable "number of pulses from background radiation" and "number of pulses induced by overvoltage". Based on the results obtained, it has been shown that these variables vary in their contributions to the expanded and combined uncertainties, in cases when the technological solution of the counter's reading system is changed.

UNCERTAINTY SOURCES OF GM COUNTERS

According to the known characteristics of a GM counter [3], by means of a process of detecting ionizing radiation, potential sources of uncertainty such as: the dependence of detection on the energy and incident angle of radiation, counter dead time, the reading system (by means of the resolution of the instrument), instrument calibration errors, influence of background radiation, uncertainty arising from the measurement process (counting impulses), influence of the overvoltage phenomenon in electronic devices (their wire structures) generated by the induction of overvoltage on the electromagnetic rays as a consequence of electromagnetic radiation in the environment where the measurements are performed (this phenomenon being especially marked in urban environments) can be indentified.

In essence, the functioning of a GM counter is based on the self-sustained avalanche gaseous effect and, in that sense, the energy of incident radiation determines the number of free, potentially initial electrons in the counter's tube, meaning that the said energy contributes significantly to the stochastic response of the counter and, in fact, to the statistical discharge time [4], directly determining the nature of a type A uncertainty. In a similar way, the angle of incident radiation contributes to type A uncertainty, because the number and position of free electrons depend on this angle, that being especially marked in a tube with a coaxial electric field. The dead time of a counter is a source of type B or combined uncertainty, depending on the determining method. Determining dead time by recording pulses at the counter's output can be conditionally arranged to suit type B uncertainty, while determining dead time by the two sources method is to be considered as a combined uncertainty, because the stochastical nature of radioactive decay has to be taken in consideration.

The applied reading system is a source of type B uncertainty and depends on the resolution of the counter's technological solution, in the same way the true value of a measured variable (the electrical discharge throughout the counter's tube which is of an analog nature) is symmetrically arranged through the digital reading, uniformly distributed over an interval of $n - 1/2$ and $n + 1/2$ digits. The uncertainty due to instrument calibration is of type B and, in almost all cases, a component of uncertainty arising from systematic effects. Background cosmic radiation is a source of combined uncertainty and the determination of this kind of uncertainty, without doubt, the most difficult one because of its fluctuation and energy structure. The contribution of background radiation to uncertainty can be decreased by applying anticoincidence protection and background radiation correction, but it can never be completely eliminated. The uncertainty of counting impulses from a radioactive source is of type A, because the deexcitation of a nucleus is completely random and nothing can determine the deexcitation moment. In other words, it is impossible to connect deexcitation with any measuring law – since this process would be of a stochastical nature and associated with the Gaussian distribution (because of the possible time balance in its occurrence).

The minimization of electronic components and, to an even greater extent, the exposure of the environment to electromagnetic radiation, lead to frequent occurrences of overvoltages within electronic devices within GM counters, which can, independently of the counter's tube, trigger the reading system and, in doing so, cause a type A uncertainty. The influence of these sources of uncertainty can be decreased by applying an overvoltage protection system of electronic devices (coordination of isolations at low voltage levels) and/or by performing measurements in an area protected from electromagnetic radiation (over 100 dB protection).

EXPERIMENTATION AND PROCESSING OF MEASUREMENT DATA

With the aim of determining measurement uncertainties of GM counters stemming from background radiation and the overvoltage phenomenon, all experiments were performed in highly-controlled laboratory conditions, involving four types of GM counters, with the same counter's tube being applied. The GM counters used were: (1) – a counter made in a discrete semiconductor technology without an anticoincidence protection, (2) – a counter made in a discrete semiconductor technology with an anticoincidence protection, (3) – a counter made in an integrated technology without an anticoincidence protection, and (4) – a counter made in tube technology without an anticoincidence protection. During the experiment, background radiation and number of pulses from the radioactive radium source were measured in the proximities of each type of the counters.

In order to statistically determine the distribution of the random variable "number of pulses from background radiation" and "number of pulses from a radioactive source", the experiments were performed in two series, involving 400 successive measurement per series, in 5 s intervals, without a radioactive source and with it, respectively. Similarly, the statistical distribution of the random variable of a number of pulses induced by overvoltage was determined, whereby, during the experiment, all of the radioactive sources were displaced from the laboratory and a two-electrode spark chambers (with a 3 mm inter-electrode gap, atmospheric conditions and a 100 kV voltage and frequency of successive breakdowns amounting to, approximately 8 Hz), was placed close to the counter. During these measurements, the counter's tube was protected from the chamber's influence by a leaded screen (made of leaded bricks) and, in that sense, only the electronic devices of the GM counter were exposed to electromagnetic radiation. The obtained results were corrected to background radiation and counter dead time (determined by the two sources method). During the experiment, other relevant parameters had constant values.

According to the obtained and corrected results, the time fluctuation of the measured variable is determined. Then the theoretical statistical distributions, assigned to a random variable "number of pulses from background radiation", "number of pulses from a radioactive source" and "number of pulses induced by overvoltage," are defined. Appropriate distribution parameters are determined by the momentum method and the maximum likelihood method with indirect likelihood function estimation [5]. It is from these results and depending on the conditions of the experiment that the possibility of determining measurement uncertainty arises.

Uncertainty arising from the counter's dead time is determined according to

$$\frac{\mathrm{d}n}{n} \quad \frac{\partial n}{\partial m}\mathrm{d}m \quad \frac{\partial n}{\partial \tau}\mathrm{d}\tau \tag{1}$$

where n is the real number of pulses, m is the registered number of pulses corrected on background radiation and τ is the dead time of the counter. According to

$$\tau \quad \frac{m_1 \quad m_2 \quad m_{12} \quad B}{m_{12}^2 \quad m_1^2 \quad m_2^2} \tag{2}$$

the dead time of the counter is calculated, where m_1 is the registered number of pulses with the first source (within the determination of dead time by the two sources method), m_2 is the registered number of pulses with the second source, m_{12} is the registered number of pulses with both of them and B is the number of pulses from background radiation, and further, according to eq. (2)

$$\frac{\mathrm{d}\tau}{\tau} \quad \frac{\partial\tau}{\partial m_1}\mathrm{d}m_1 \quad \frac{\partial\tau}{\partial m_2}\mathrm{d}m_2 \quad \frac{\partial\tau}{\partial m_{12}}\mathrm{d}m_{12} \quad \frac{\partial\tau}{\partial B}\mathrm{d}B \tag{3}$$

and

$$\begin{gathered}\mathrm{d}m_1 \quad m_1 u_{\mathrm{m1A}},\ \mathrm{d}m_2 \quad m_2 u_{\mathrm{m2A}}, \\ \mathrm{d}m_{12} \quad m_{12} u_{\mathrm{m12A}},\ \mathrm{d}B \quad B u_{\mathrm{BA}}\end{gathered} \tag{4}$$

where u_{m1A}, u_{m2A}, u_{m12A} are the uncertainties of type A arising from registering pulses with first, second, and both sources simultaneously, and u_{BA} is the uncertainty of type A arising from background radiation.

The uncertainty of the reading system arising from the resolution of the measuring instrument, given the previous hypothesis that the real value of measured variables exists over the interval of n – 1/2 to n + 1/2 digits with a uniform probability, where the resolution has a value of a single digit in all measurements, yields to the expanded uncertainty with a value of 0.5 digits. The standard deviation, according to the uniform distribution assigned to the uncertainty by the resolution of the reading system, is $\sigma = 0.5/3^{1/2}$ and standard uncertainty is u_{B} = 0.29 digits [6].

As it has already been said, background cosmic radiation is a source of combined uncertainty, but this applies only to the type A uncertainty which is determined according to the experimentally established fact that the random variable "number of pulses from background radiation" belongs to the Poisson distribution. The second key moment pertaining to this empirical distribution is determined as a product of the number of events and appropriate event probability. The histogram and probability density function, figs. 2 and 3, respectively, represent experimentally obtained results for a GM counter made in an integrated technology without anticoincidence protection.

We have offered a theoretical explanation, backed by experimental results, that the random variable "number of pulses from a radioactive source" belongs to the Gaussian distribution. We have, also, determined the second key moment of consequence for the standard uncertainty of type A. The histogram and probability density function, figs. 4 and 5, respectively, represent experimentally obtained results for a GM counter made in an integrated technology without anticoincidence protection.

The uncertainty caused by the overvoltage phenomenon is of type A and it has been experimentally established that the random variable "number of pulses induced by overvoltage" belong to the Poisson distribution. The uncertainty caused by overvoltage is determined in the same way as the uncertainty caused by background radiation. The histogram and probability density function, figs. 6 and 7, respectively, represent experimentally obtained results for a GM counter made in an integrated technology without anticoincidence protection.

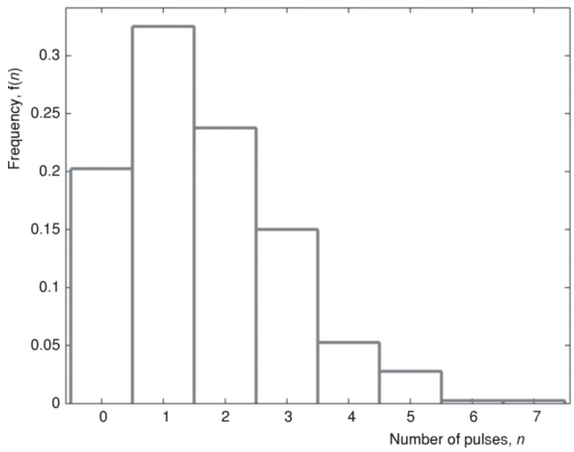

Figure 2. Histogram of background radiation number of pulses in the case of a GM counter made in an integrated technology without anticoincidence protection

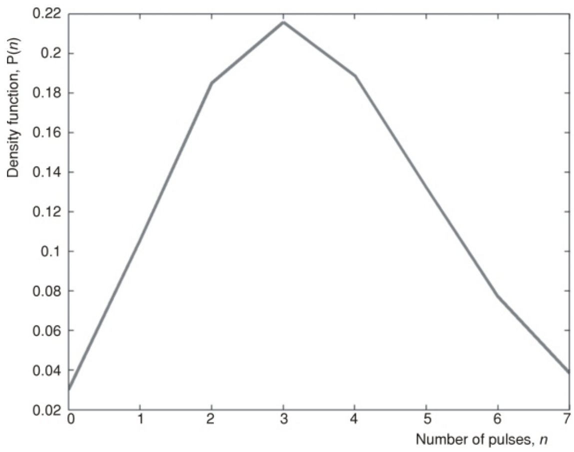

Figure 3. Density function of background radiation number of pulses in the case of a GM counter made in an integrated technology without anticoincidence protection

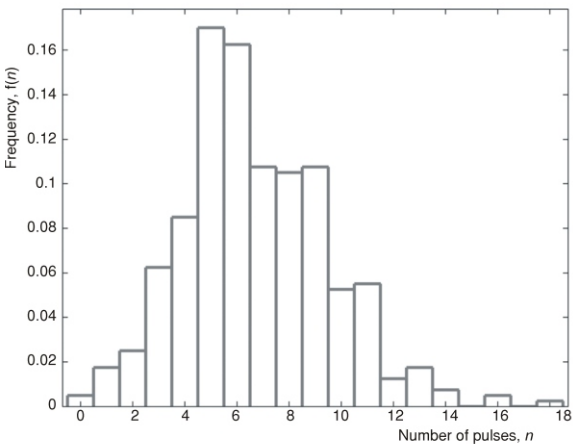

Figure 4. Histogram of the radioactive source of number of pulses in the case of a GM counter made in an integrated technology without anticoincidence protection

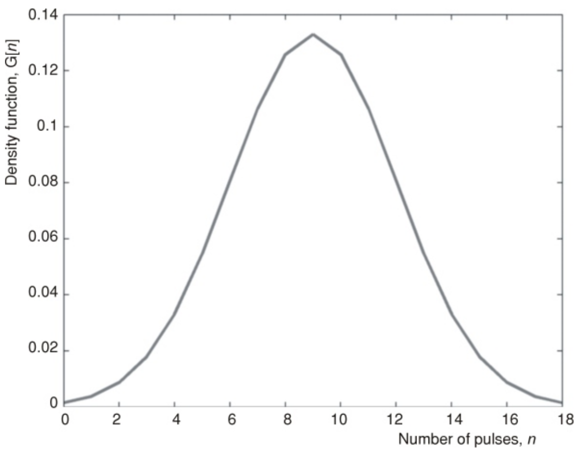

Figure 5. Density function of a radioactive source of a number of pulses in the case of a GM counter made in an integrated technology without anticoincidence protection

Type A and type B uncertainties obtained by previously explained procedure are conditionally independent of each other and standard combined uncertainty can be determined as [2]

$$u_c = \sqrt{u_{\mathrm{A}}^2 + u_{\mathrm{B}}^2} \quad (5)$$

The expanded and combined uncertainty (overall uncertainty) is determined by multiplying eq. (5) and coverage factor *k*. In the case of type A and type B uncertainties, this factor gains its values over the interval ($3^{1/2}$, 3), depending on the assigned distribution. In the case of a combined uncertainty, as a compromise for the value of the coverage factor, the value of 2.5 can be adopted [1].

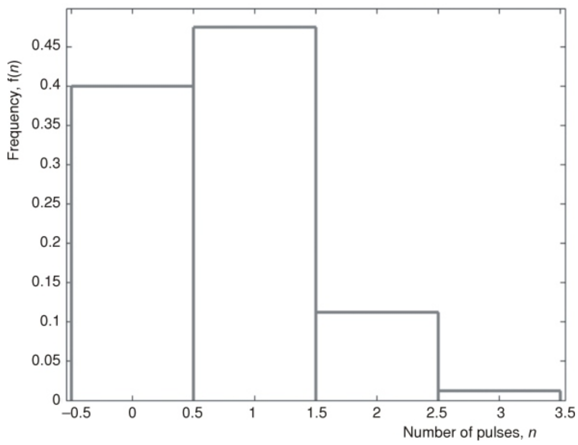

Figure 6. Histogram of induced overvoltage number of pulses in the case of a GM counter made in integrated technology without anticoincidence protection

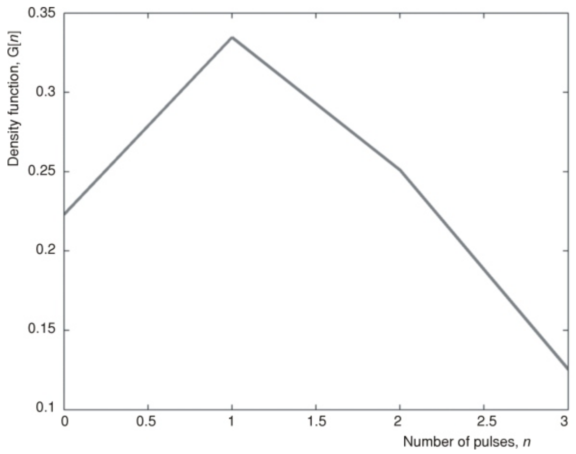

Figure 7. Density function of an induced overvoltage number of pulses in the case of a GM counter made in an integrated technology without anticoincidence protection

RESULTS AND DISCUSSION

The overall uncertainty for a counter made in a discrete semiconductor technology without anticoincidence protection is 12.31%.

Overall uncertainty for a counter made in a discrete semiconductor technology with anticoincidence protection is 6.93%.

Overall uncertainty for a counter made in an integrated technology without anticoincidence protection is 17.16%.

Overall uncertainty for a counter made in tube technology without anticoincidence protection is 9.24%.

The major contributors of the differences between overall uncertainty values are the uncertainties caused by background radiation and induced overvoltages. As for the counters made in tube technology, the uncertainty caused by induced overvoltages has not been noticed, as opposed to the cases of counters made in an integrated technology where the said source of uncertainty is a major contributor of overall uncertainty. In cases involving counters with an anticoincidence protection, the uncertainty caused by background radiation has not been noticed. For all other types of counters, this source of uncertainty has proved to be of an approximately same value. Other known sources of uncertainty did not significantly differ among themselves.

CONCLUSION

Based on considerations previously stated, the influence of cosmic background radiation and overvoltage phenomena on the value of expanded and combined uncertainty in measurements with different types of GM counters is shown here. The minimization of electronic components and, even to a greater extent, the exposure of environment to electromagnetic radiation, lead to frequent occurrences of overvoltages within the electronic devices of GM counters which can trigger the reading system.

Results obtained by a GM counter with a reading system made in an integrated technology show the influence of this phenomenon on expanded and combined uncertainty. The influence of this phenomenon on measurement uncertainty can be decreased by applying overvoltage protection of electronic devices (co-ordination of isolations at low voltage levels) and/or by performing measurements in an area protected from electromagnetic radiation (over 100 dB protection). On the other hand, the contribution of cosmic background radiation to measurement uncertainty is noticed within GM counters without anticoincidence protection.

Thus, it can be concluded that decreasing the expanded and combined uncertainty can be achieved by a suitable technological solution, *i. e.* by means of increasing both efficiency and anticoincidence and overvoltage protection of the electronic devices. In further investigations, the effects of incident radiation energy and angle on expanded and combined uncertainty are to be included, especially in the vicinity of a counter's discrimination level.

APPENDIX A

The Poisson distribution is derived from the binomial distribution when n tends toward infinity and, at the same time, $np = \lambda$ remains constant (Poisson's limiting-value statement)

$$\lim_{\substack{n \to \infty \\ np = \lambda}} \binom{n}{k} p^k (1-p)^{n-k} = \frac{\lambda^k}{k!} \mathrm{e}^{-\lambda}, k = 0, 1, 2 \ldots \quad \text{(A.1)}$$

Since probability p is very small for the large n and $np = \lambda =$ constant, Poisson's distribution describes rare events.

Poisson's density function is given by

$$P(x = k) = \mathrm{e}^{-\lambda} \frac{\lambda^k}{k!}, k = 0, 1, 2 \ldots \quad \text{(A.2)}$$

The Poisson distribution is a one parameter discrete distribution that takes on nonnegative integer values. The parameter λ, is both the mean and the variance of the distribution. Thus, as the size of the numbers in a particular sample of a Poisson random number gets larger, so does the variability of that number.

The Poisson distribution is appropriate for all applications involving counting the number of times a random event occurs in a given amount of time, distance, area, *etc*. It is also used in the reliability theory and in the theory of elementary particles.

APPENDIX B

A random process produces a normally distributed variate when the latter can be conceived as the sum of a large number of independent, randomly distributed variates, and when each of these variates makes only an insignificant contribution to the sum (central limiting-value statement). This model, which can be applied to many random phenomena (including discharge processes, measuring errors and so on), brings us to the extraordinary significance of the normal distribution, derived from de Moivre, Laplace, and Gauss error and compensation calculations.

The Gaussian density function is given by

$$\varphi(x, \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} \mathrm{e}^{-(x-\mu)^2/2\sigma^2} \quad \text{(B.1)}$$

and the Gaussian distribution function is given by

$$\Phi(x, \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{x} \mathrm{e}^{-(t-\mu)^2/2\sigma^2} \mathrm{d}t \quad \text{(B.2)}$$

The Gaussian distribution is a two parameter family of curves. The first parameter, μ, is the mean, the second, σ, the standard deviation. For experimentally obtained data, the standard deviation can be derived from the mean value of the sample, according to:

$$\sigma = \sqrt{\mu} \quad \text{(B.3)}$$

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ПРОШИРЕНА И КОМБИНОВАНА МЕРНА НЕСИГУРНОСТ ГАЈГЕР-МИЛЕРОВОГ БРОЈАЧА

У раду је описан поступак добијања проширене и комбиноване мерне несигурности за четири типа ГМ бројача са истом бројачком цеви у случајевима када мерну несигурност у мерења уносе позадинско зрачење – фон и индуковане пренапонске појаве. Као последица електромагнетног зрачења, пренапонске појаве су посебно изражене у урбаним срединама. На основу добијених експерименталних резултата утврђено је да у зависности од технолошког решења бројача варирају мерне несигурности утицајних случајних величина „број импулса од позадинског зрачења” и „број импулса од пренапона” и из тог разлога дају различит допринос проширеној и комбинованој мерној несигурности код примењених типова ГМ бројача.

Кључне речи: мерна несигурност, ГМ бројач, позадинско зрачење, пренапонске појаве, мртво време