THE INFLUENCE OF THE IRON SHIELD OF THE SOLENOID ON SPIN TRACKING

by

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The influence of the iron shield of the solenoid on spin tracking is studied in this paper. In the case of the 200 MeV proton, the study has been numerically done in the ZGOUBI code. The distribution of the magnetic field was done by POISSON. We have come to the conclusion that the influence of the solenoid's shielding on spin tracking is the same at its entrance and exit and that is directly proportional to the intensity of the magnetic induction B on the axis of the solenoid. We have also determined that the influence of the solenoid's shielding is much stronger on transversal components of the spin than on its longitudinal component. The differences between components of the spin for the shielded and not-shielded solenoid diminish with the increase in the distance from the solenoid.

INTRODUCTION

In many situations, solenoids are used to capture and contain a beam of particles. A number of papers in which the solenoids are being studied (see, for example, [1-4]) already exist. The solenoid may be used to rotate the spin of a particle, the so called "spin rotator". Many papers on spin dynamics and tracking (see, for example, [5, 6]) have also been published. In practice, this gives rise to questions such as: *how strong is the influence of the solenoid's shield on spin tracking*? The result worth reporting is an explicit answer to the previous question. This paper deals with spin tracking through the shielded and non-shielded solenoid, in other words, with the influence of fringe-field shaping of the solenoid on spin tracking. In both cases, the distribution of the magnetic fields, the shielded and not-shielded one, is done in POISSON [7]. In order to determine how the intensity of the magnetic induction B on the axis of the solenoid influ-

ences the differences in spin tracking of the shielded and not-shielded solenoid, two cases of the intensity of the magnetic induction B were taken into consideration; the first one being $B = 1.30$ T, the second $B = 4.00$ T. In the case of the 200 MeV proton, the numerical analysis was done by using the ZGOUBI code [8]. Our findings lead us to the conclusion that in the case of $\varepsilon_y = \varepsilon_z = 10$ mm mrad for the beam rms emitions in the transversal planes (x axis is the longitudinal direction) and 0.5% momentum spread at half width.

THEORY

Spin tracking

The theory of spin motion is described in [5, 6]. Here we repeat it briefly.

The motion of the spin $\vec{S}$ is governed by the Thomas-BMT first order differential equation:

$$\frac{d\vec{S}}{dt} = \frac{q}{\gamma m}\vec{S} \times \vec{\Omega} \qquad (1)$$

where $\vec{\Omega}$ is the spin precession given by:

$$\vec{\Omega} = (1 + \gamma G)\vec{b} + G(1 + \gamma)\vec{b}_{\parallel} \qquad (2)$$

where q is the charge of the particle, m – the rest mass of the particle, γ – the Lorentz relativistic factor, G – the anomalous magnetic moment of the

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particle [5, 6, 8] (for the proton $G = 1.7928$), $\vec{b}$ – the magnetic field, $\vec{b}_\parallel$ – the component of $\vec{b}$ which is parallel to the velocity $\vec{v}$ of the particle.

Let $b = |\vec{b}|$ and $v = |\vec{v}|$; $ds = v\,dt$ is the differential path, $B\rho = \gamma mv/q$ is the rigidity of the particle.

Introducing the derivative of the spin with respect to the path:

$$\vec{S}' = \frac{d\vec{S}}{ds} = \frac{1}{v}\frac{d\vec{S}}{dt} \qquad (3)$$

as well as $\vec{B} = \vec{b}/B\rho$ and $\vec{B}_\parallel = \vec{b}_\parallel/B\rho$, and

$$\vec{\omega} = \frac{\vec{\Omega}}{B\rho} = (1+\gamma G)\vec{B} + G(1+\gamma)\vec{B}_\parallel \qquad (4)$$

eq. (1) can be rewritten as follows [6, 8]:

$$\vec{S}' = \vec{S} \times \vec{\omega} \qquad (5)$$

This equation is then solved using a highly symplectic numerical method based on the truncated Taylors series [6, 8].

Numerical method

The symplectic numerical method is based on the resolution of the equation by means of the Taylor expansion. In the case of eq. (5), from the values of the magnetic factor $\vec{\omega}(M_0)$ and the spin $\vec{S}(M_0)$ of the particle at position M_0 of its trajectory, the spin $\vec{S}(M_1)$ at position M_1, following a displacement ds, is given by the Taylor expansion:

$$\vec{S}(M_1) = \vec{S}(M_0) + \frac{d\vec{S}}{ds}(M_0)ds + \frac{d^2\vec{S}}{ds^2}(M_0)\frac{ds^2}{2}$$

$$+ \frac{d^3\vec{S}}{ds^3}(M_0)\frac{ds^3}{3!} + \frac{d^4\vec{S}}{ds^4}(M_0)\frac{ds^4}{4!} \qquad (6)$$

The derivatives $\vec{S}^{(n)} = \frac{d^n\vec{S}}{ds^n}$ of $\vec{S}$ at M_0 are obtained by differentiating eq. (5)

$$\vec{S}' = \vec{S} \times \vec{\omega} \qquad (7a)$$

$$\vec{S}'' = \vec{S} \times \vec{\omega}' + \vec{S}' \times \vec{\omega} \qquad (7b)$$

$$\vec{S}''' = \vec{S} \times \vec{\omega}'' + 2\vec{S}' \times \vec{\omega}' + \vec{S}'' \times \vec{\omega} \qquad (7c)$$

$$\vec{S}^{IV} = \vec{S} \times \vec{\omega}''' + 3\vec{S}' \times \vec{\omega}'' + 3\vec{S}'' \times \vec{\omega}' + \vec{S}''' \times \vec{\omega} \qquad (7d)$$

where the derivatives $\vec{\omega}^{(n)} = d^n\vec{\omega}/ds^n$ are obtained from eq. (4).

The next step is getting $\vec{B}_\parallel$ and its derivatives. This can be done in the following way. Let $\vec{u} = \vec{v}/v$ be the normalized velocity of the particle, then:

$$\vec{B}_\parallel = (\vec{B}\cdot\vec{u})\vec{u} \qquad (8a)$$

$$\vec{B}_\parallel' = (\vec{B}'\cdot\vec{u} + \vec{B}\cdot\vec{u}')\vec{u} + (\vec{B}\cdot\vec{u})\vec{u}' \qquad (8b)$$

$$\vec{B}_\parallel'' = (\vec{B}''\cdot\vec{u} + 2\vec{B}'\cdot\vec{u}' + \vec{B}\cdot\vec{u}'')\vec{u}$$
$$+ 2(\vec{B}'\cdot\vec{u} + \vec{B}\cdot\vec{u}')\vec{u}' + (\vec{B}\cdot\vec{u})\vec{u}'' \qquad (8c)$$

etc. where $\vec{B}_\parallel^{(n)} = d^n\vec{B}_\parallel/ds^n$.

The last point consists in geting $\vec{u}$, $\vec{B}$, and their n^{th} derivatives. This can be done by means of the Lorentz equation which governs the motion of a charged particle q, mass m and velocity $\vec{v}$ in magnetic field $\vec{b}$:

$$\frac{d(m\vec{v})}{dt} = q\vec{v} \times \vec{b} \qquad (9)$$

Taking $\vec{u} = \vec{v}/v$, $ds = v\,dt$, $\vec{u}' = d\vec{u}/ds$, $m\vec{v} = mv\vec{u} = qB\rho\vec{u}$ where $B\rho$ is the rigidity of the particle, $\vec{B} = \vec{b}/B\rho$, the Lorentz equation can be rewritten:

$$\vec{u}' = \vec{u} \times \vec{B} \qquad (10a)$$

From eq. (10a) we have:

$$\vec{u}'' = \vec{u}' \times \vec{B} + \vec{u} \times \vec{B}' \qquad (10b)$$

$$\vec{u}''' = \vec{u}'' \times \vec{B} + 2\vec{u}' \times \vec{B}' + \vec{u} \times \vec{B}'' \qquad (10c)$$

$$\vec{u}^{IV} = \vec{u}''' \times \vec{B} + 3\vec{u}'' \times \vec{B}'$$
$$+ 3\vec{u}' \times \vec{B}'' + \vec{u} \times \vec{B}''' \qquad (10d)$$

where $\vec{B}^{(n)} = d^n\vec{B}/ds^n$.

From $d\vec{B} = \frac{\partial\vec{B}}{\partial X}dX + \frac{\partial\vec{B}}{\partial Y}dY + \frac{\partial\vec{B}}{\partial Z}dZ = \sum_{i=1}^{3}\frac{\partial\vec{B}}{\partial X_i}dX_i$ (see [6, 8]) we get:

$$\vec{B}' = \sum_{i=1}^{3}\frac{\partial\vec{B}}{\partial X_i}u_i \qquad (11a)$$

$$\vec{B}'' = \sum_{i,j=1}^{3}\frac{\partial^2\vec{B}}{\partial X_i\,\partial X_j}u_iu_j + \sum_{i=1}^{3}\frac{\partial\vec{B}}{\partial X_i}u_i' \qquad (11b)$$

$$\vec{B}''' = \sum_{i,j,k=1}^{3}\frac{\partial^3\vec{B}}{\partial X_i\,\partial X_j\,\partial X_k}u_iu_ju_k$$

$$+ 3\sum_{i,j=1}^{3}\frac{\partial^2\vec{B}}{\partial X_i\,\partial X_j}u_iu_j' + \sum_{i=1}^{3}\frac{\partial\vec{B}}{\partial X_i}u_i'' \qquad (11c)$$

$$\vec{B}^{IV} = \sum_{i,j,k,l=1}^{3} \frac{\partial^4 \vec{B}}{\partial X_i \, \partial X_j \, \partial X_k \, \partial X_l} u_i u_j u_k u_l$$

$$6 \sum_{i,j,k=1}^{3} \frac{\partial^3 \vec{B}}{\partial X_i \, \partial X_j \, \partial X_k} u_i u_j u_k$$

$$4 \sum_{i,j=1}^{3} \frac{\partial^2 \vec{B}}{\partial X_i \, \partial X_j} u_i u_j$$

$$3 \sum_{i,j=1}^{3} \frac{\partial^2 \vec{B}}{\partial X_i \, \partial X_j} u_i u_j \quad \sum_{i=1}^{3} \frac{\partial \vec{B}}{\partial X_i} u_i \quad (11d)$$

Now, using eqs. (4), (7), (8), (10), and (11), eq. (6) can be solved numerically.

RESULTS

Magnetic field distribution

The numerical results presented here regard a solenoid of a length of $L = 80$ cm and with the Bore radius of $R_b = 8$ cm . Other dimensions regarding the shielded solenoid are presented in fig. 1.

The distribution of the magnetic fields, calculated by means of POISSON with the mesh spacing of 1 mm, are presented in fig. 2, in the case when the solenoid is shielded (dotted line) and when the solenoid is not shielded (full line).

Spin tracking

This analysis has been carried out by means of the ZGOUBI code in the case of the 200 MeV proton beam of $\varepsilon_y = \varepsilon_z = $ mm mrad for the beam rms emittances in the transversal planes (x axis is the longitudinal direction) and 0.5% momentum spread at half width. We have considered the case when the starting value of one of the *transversal* components of the spin equals one (for example, $S_x^{(0)} \ 0, S_y^{(0)} \ 1, S_z^{(0)} \ 0$, see fig. 3), as well as the one when the starting value of the *longitudinal* component of the spin equaling one ($S_x^{(0)} \ 1, S_y^{(0)} \ 0, S_z^{(0)} \ 0$, see fig. 4). In figs. 3 and 4 the variables ΔS_x , ΔS_y ,and ΔS_z, are defined as the differences between corresponding components of the spin for the shielded and not-shielded solenoid.

From figs. 3 and 4 it can be deduced that the influence of the solenoid's shielding on spin tracking does not differ at its entrance and exit. It is also possible to conclude that the influence of the
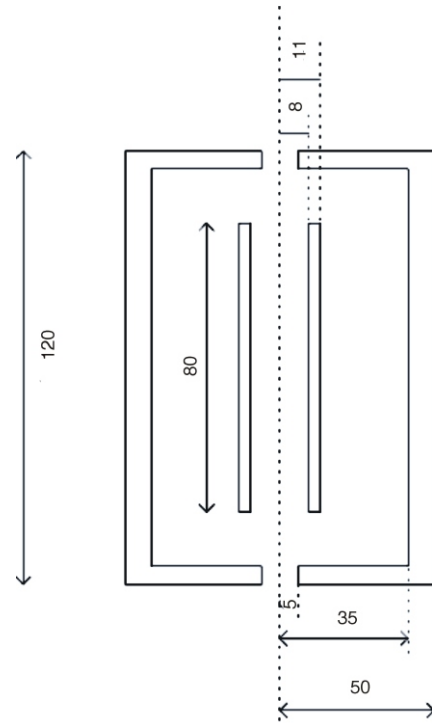

Figure 1. Sketch of the shielded solenoid. The dimensions are in centimeters

solenoid's shielding on spin tracking is directly proportional to the intensity of the magnetic induction B on the axis of the solenoid and that the influence of the solenoid's shielding is much stronger on the transversal components of the spin compared to the longitudinal one. The differences between components of the spin for the shielded and not-shielded solenoid fade with the increase of the distance from the solenoid.
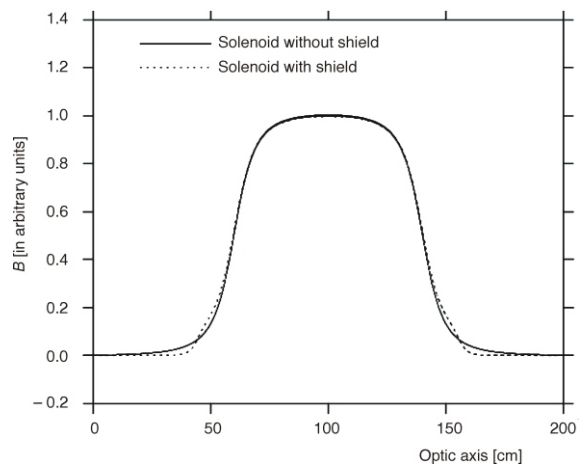

Figure 2. Distribution of the magnetic fields, calculated by POISSON, on the axis of the solenoid in the case when the solenoid is shielded (dotted line) and when the solenoid is not shielded (full line)
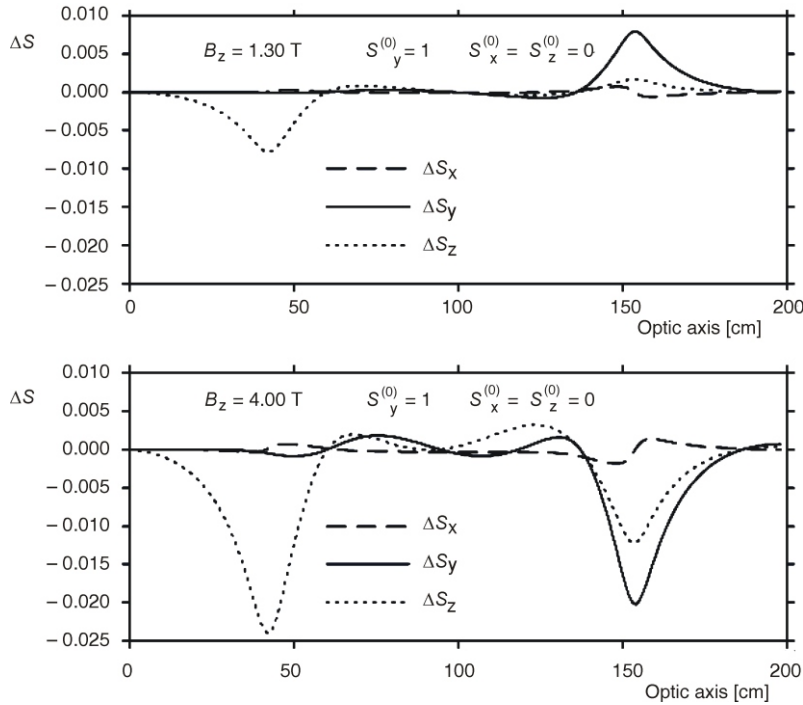
Figure 3. Differences between components of the spin for the shielded and not-shielded solenoid in the case when the starting value of one of the *transversal* components of the spin is equal one (for example $S_x^{(0)} = 0$, $S_y^{(0)} = 1$, $S_z^{(0)} = 0$). The intensity of the magnetic induction *B* on the axis of the solenoid is 1.30 T (up) and 4.00 T (low). The other marks are explained in the text
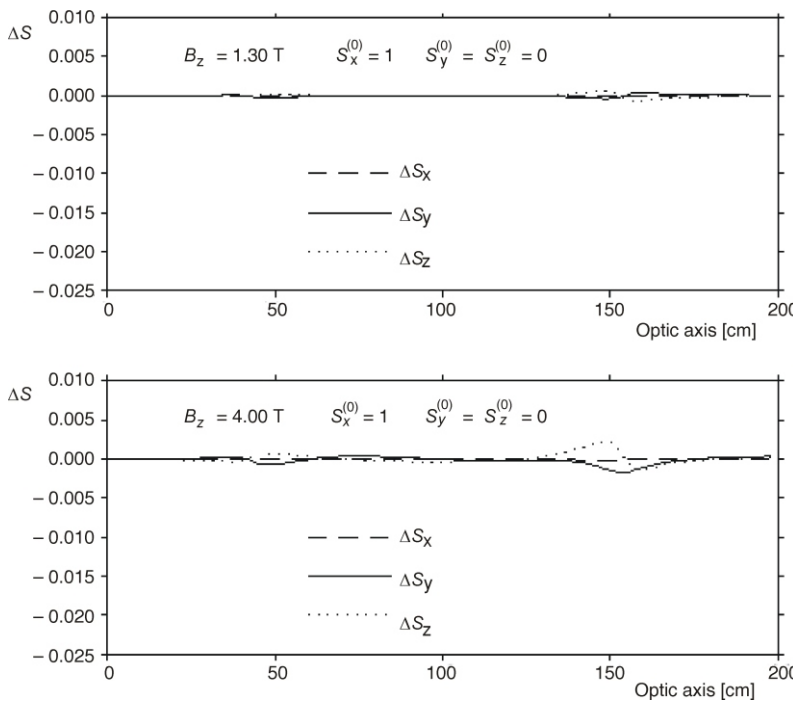

Figure 4. Differences between components of the spin for the shielded and not-shielded solenoid in the case when the starting value of the *longitudinal* components of the spin is equal one ($S_x^{(0)} = 1$, $S_y^{(0)} = 0$, $S_z^{(0)} = 0$). The intensity of the magnetic induction *B* on the axis of the solenoid is 1.30 T (up) and 4.00 T (low). The other marks are explained in the text

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УТИЦАЈ ГВОЗДЕНОГ КУЋИШТА СОЛЕНОИДА НА ОРИЈЕНТАЦИЈУ СПИНА

У раду је разматран утицај гвозденог кућишта соленоида на оријентацију спина наелектрисане честице која се креће кроз соленоид. Резултати су добијени коришћењем ZGOUBI програма, који користи нумеричке методе засноване на развоју у Тејлоров ред, у случају протона енергије 200 MeV. Дистрибуција магнетног поља дуж соленоида добијена је коришћењем програма POISSON. Утицај гвозденог кућишта соленоида на оријентацију спина је иста, како на улазу, тако и на излазу из соленоида и директно је пропорционална интензитету магнетне индукције дуж осе соленоида. Утицај кућишта соленоида је много јачи на трансверзалне компоненте спина него на лонгитудиналну компоненту. Разлика у оријентацији спина у случају соленоида са и без кућишта опада са повећањем растојања дуж осе соленоида.

Кључне речи: спин, соленоид, ефекат крајева