INFLUENCE OF SPIN ON FISSION FRAGMENTS ANISOTROPY

by

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An analysis of selected fission fragment angular distribution when at least one of the spins of the projectile or target is appreciable in induced fission was made by using the statistical scission model. The results of this model predicate that the spins of the projectile or target are affected on the nuclear level density of the compound nucleus. The experimental data was analysed by means of the couple channel spin effect formalism. This formalism suggests that the projectile spin is more effective on angular anisotropies within the limits of energy near the fusion barrier.

Key words: couple channel spin effect, fusion-fission reaction, angular distribution anisotropies

INTRODUCTION

A large amount of experimental information on fission fragment angular distribution for projectile induced fission has been published. In most cases, these experimental results can be explained within the framework of the entrance channel dependent (ECD) K-state distribution model [1, 2], with the exclusion of target projectile ground state spins. For reactions in which the spins of the projectile and target are appreciable, it is expected that the angular anisotropy will be affected by the inclusion of spins. In order to investigate the importance of this effect on angular anisotropies, we have employed the couple channel spin formalism. Within this framework, the partial probability of the formation of a state (I, M) of the compound nucleus depends on J value, in spite of the relative angular distribution l. The quantum number J equals the summation of the projectile spin S with the relative angular distribution l. We have also analyzed the fission fragment angular distribution by using the statistical scission model (SSM). Versions of this model have been published by various authors [3-5].

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In the folowing two sections, the theoretical formulas for considering the couple channel spin effect and the basic SSM formula are presented, respectively. Results obtained from the couple channel spin effect and SSM are compared with experimental data and discussed in the last section.

THE COUPLE CHANNEL SPIN FORMALISM

The fissioning-transition nucleus is wholly characterized by the quantum numbers I (total angular momentum), M (projection of I on the space-fixed axis to be designated as the projectile-beam direction), and K (projection of I on the nuclear-symmetry axis). If it is assumed that the fragments separate along the nuclear symmetry axis, the angular distribution is uniquely determined by the above quantum numbers [6, 7].

If a compound state (I, M) fissions through a transition state K, the angular distribution is given by the square of the rotational part of the collective wave function:

$$W^{I}_{M,K}(\theta) \quad \frac{(2I \quad 1)}{4\pi}\left|\mathrm{d}^{I}_{M,K}(\theta)\right|^{2} \tag{1}$$

The normalized $d^{I}_{M,K}(\theta)$ functions are defined by [8]:

$$d^{I}_{M,K}(\theta) \quad [(I \quad M)!(I \quad M)!(I \quad K)!(I \quad K)!]^{1/2}$$

$$\frac{(\quad 1)^{x}\left(\frac{\sin\theta}{2}\right)^{K \ M \ 2x}\left(\frac{\cos\theta}{2}\right)^{2I \ K \ M \ 2x}}{{}_{x}\,(I \quad K \quad x)!(I \quad M \quad x)!(x \quad K \quad M)!x!} \tag{2}$$

where the sum runs over $x = 0, 1, 2, \ldots$ and contains all terms in which no negative value appears in the denominator of the sum for any of the quantities in parentheses. Therefore, the fission-fragment angular distribution offers a direct source of information on the spectrum of quantum states associated with the transition nucleus.

The relative fission-fragment angular cross section is given by [8]:

$$W(\theta) = \sum_{j}\sum_{J}\sum_{M} P(j,J,M)W(J,M,\theta) \tag{3}$$

the first term being a partial probability of the formation of a state (I, M) of the compound nucleus from a particular J value [9]:

$$P(J;I,M) = \sum_{l_{min}}^{l_{max}} P(l,J;I,M) \tag{4}$$

where

$$P(l,J;I,M) = \frac{\sum_{\sigma,\mu}(2l+1)T_l^J(E)\left|C_{0,\sigma,\sigma}^{l,S,J}\right|^2\left|C_{\sigma,\mu,M}^{J,I_0,I}\right|^2}{(2I_0+1)\sum_{J,l,\sigma}(2l+1)T_l^J(E)\left|C_{0,\sigma,\sigma}^{l,S,J}\right|^2} \tag{5}$$

transmission coefficients $T_l^J(E)$ are derived from the optical model with the spin-orbit interaction, where $J = l + S$ and S is the projectile spin with projection σ on the space-fixed axis, both denoted by I_0 and μ, respectively. The total angular momentum I of the compound nucleus is given by $I = J + I_0$, and the projection of I on the space-fixed axis is given by M. The term $C_{\sigma,\mu,M}^{J,I_0,I}$ is a Clebsch-Gordan coefficient.

The second term $W(I,M;\theta)$ in eq. (3) gives the angular distribution of the fission fragments emitted from the compound state (I, M):

$$W(I,M;\theta) = \frac{\sum_{K=-I}^{I}\exp\left(-\frac{K^2}{2K_0^2}\right)\left(\frac{2I+1}{2}\right)\left|d_{M,K}^{I}(\theta)\right|^2}{\sum_{K=-I}^{J}\exp\left(-\frac{K^2}{2K_0^2}\right)} \tag{6}$$

also, excited levels in the transition nucleus are described by statistical theory. The K-distribution of these levels is predicted to be Gaussian by Halpen *et al.* [10]:

$$F(K) = \exp\left(-\frac{K^2}{2K_0^2}\right) \tag{7}$$

with a variance of

$$K_0^2 = \frac{J_{eff}T}{\hbar^2} \tag{8}$$

the effective moment of inertia is $J_{eff} = J_{\|}J_{\perp}/(J_{\|} - J_{\perp})$ where $J_{\perp}$ and $J_{\|}$ are nuclear moments of inertia about the axis perpendicular and parallel to the symmetry axis, respectively, and T is the temperature of the nucleus in the transition state.

FORMALISM OF THE STATISTICAL SCISSION MODEL

According to the SSM, the relative cross-section, $W(\theta)$, for fission fragments to be emitted in direction $\hat{n}$ forming an angle θ with the beam axis, when the target projectile spins are zero, is given by Huizenga *et al.* [11]:

$$W(\theta) = \sum_{I=I_{\min}}^{I_{\max}}(2I+1)T_I$$

$$\sum_{K=-I}^{I}\left(\frac{2I+1}{2}\right)\left|d_{M=0,K}^{J}(\theta)\right|^2\frac{\exp\left(-\frac{S^2}{2S_0^2}\right)}{\sum_{K=-I}^{I}\exp\left(-\frac{S^2}{2S_0^2}\right)} \tag{9}$$

Here again the distribution of spin projection M (the projection of the total angular momentum I along $\hat{n}$) is taken to be a Gaussian with a variance of S_0^2, where S_0^2 for spherical fission fragment is given by either of the following equations [12]:

$$S_0^2 = 2I_{\text{sph}}\frac{T}{\hbar^2}\frac{2I_{\text{sph}}+\mu R_c^2}{\mu R_c^2} \tag{10}$$

or

$$S_0^2 = 2\sigma^2\frac{2\sigma^2+\mu R_c^2\frac{T}{\hbar^2}}{\mu R_c^2\frac{T}{\hbar^2}} \tag{11}$$

with $\sigma^2 = I_{\text{sph}}T/\hbar^2 = (2/5)MR^2T/\hbar^2$. The quantities I_{sph}, T, M, and R are the moments of inertia, nuclear temperature, mass and radius of one of the symmetric fission fragments, while R_c is the distance between the centers of fragments at scission configuration and equals $1.225(A_1^{1/3}+A_1^{1/3})(c/a)^{2/3}$ (A_1 and A_2 are mass numbers of fission fragments).

For a scission configuration of two unattached deformed fragments, the variance S_0^2 is given by either of the two equations [12]:

$$S_0^2 = \frac{2I_{\|}\frac{T}{\hbar^2}}{\frac{2I_{\perp}+\mu R_c^2}{2I_{\perp}+\mu R_c^2+2I_{\|}}} \tag{12}$$

or

$$S_0^2 = 2\sigma_{\|}^2\frac{2\sigma_{\perp}^2+\mu R_c^2\frac{T}{\hbar^2}}{2\sigma_{\perp}^2+\mu R_c^2\frac{T}{\hbar^2}+2\sigma_{\|}^2} \tag{13}$$

where $I_{\|}, I \quad , \sigma_{\|}^{2}$, and σ^2 are moments of inertia and spin cutoff parameters of a single fission fragment rotating about an axis parallel and perpendicular to the symmetry axis, respectively.

The primary fission fragments are assumed to have non-spheroid shapes, with the principal one-half axes of magnitude, in terms of their ratio c/a, namely $a = 1.225(A_c/2)^{1/3}(c/a)^{-1/3}$ and $c = {=}1.225(A_c/2)^{1/3}(c/a)^{2/3}$ where A_c is the mass number of a composite system. The total intrinsic excitation energy of the two fission fragments at scission is given by [13]:

$$E^{*} \quad E_{c.m} \quad Q \quad E_{\text{Coul}} \quad E_{\text{def}} \quad E_{\text{rot}} \tag{14}$$

where Q represents the difference in energy between the entrance channel nuclei and the ground state of the two fission fragments. Here, $E_{\text{Coul}} + E_{\text{def}}$ is the sum of the Coulomb and deformation energies stored in the potential energy at the instant scission. The Coulomb energy is estimated by means of the expression [13]:

$$E_{\text{Coul}}\,[\text{MeV}] \quad 1.44\,\frac{Z^2}{2c} \tag{15}$$

where Z is one-half of the charge of the composite system. The rotational energy E_{rot} of the system at the scission configuration for spin I and projection m on the scission axis is [14]:

$$E_{\text{rot}} \quad \frac{I \quad \frac{1}{2}^{\;2} \quad m^2 \;\; \hbar^2}{2\mu c^2 \quad 4I} \tag{16}$$

where μ is the reduced mass of the fission fragments. The temperature of each fission fragment is assumed to be given by:

$$T \quad \frac{\frac{E^{*}}{2}}{\text{LDP}}^{\;1/2} \tag{17}$$

where LDP is the liquid drop parameter, A is the mass number of one fragment, and the total excitation energy E^{*} is divided equally between the two symmetric fission fragments.

RESULTS AND DISCUSSION

In order to investigate the effect of spin on the angular distribution of fragments in induced fission, we have considered the reactions in which the projectile or target spins, or both, are considerable. These reactions are shown in tab. 1. The experimental data are obtained form references [15] and [16]. Since the energy of the projectile in these reactions is an approximation of the order of a fusion barrier, the SSM is a suitable model for calculating the angular distribution of fission fragments [17]. The fusion barrier itself has been calculated by means of semi-classical formalism [18]. The results of SSM with $E_{\text{d}} = 20$ and tow different choices for LDP equal to $A/8$ and $A/20$ (shown in tab. 1). The excitation energy and temperature of the nucleus undergoing the fission process are also set in this table. The SSM predications for the angular distribution of fission fragments for different LDP tow choices are illustrated in figs. 1-6. These figures show that the results of SSM with LDP = $A/20$ are in good agreement with experimental results, contrary to the results obtained with LDP = $A/8$. Actually, angular anisotropies of such reactions depend on the suitable rate of the LDP parameter. Since this parameter is related to the density levels of the compound nucleus in fission point, SSM suggests that anisotropies in reactions in which the value of the target and projectile spins are considerable, depend on the level of the density of the nucleus in fission point. It is important to note that such a consideration cannot describe the effect of projectile and target spins on level density. For this reason, we have employed the couple channel spin effect formalism. This formalism is based on the coupling of the spin of the projectile S and the relative angular momentum l. Also, this formalism allows us to investigate the effect of projectile and target spins on angular anisotropies, since parameter is related to the level

Table 1. Variances obtained from theoretical calculation

Reaction	E_{lab} [MeV]	$\frac{E_{\text{lab}}}{B_{\text{fus}}}$	E^{*} [MeV]	T [MeV]	$S_0^{2\,(a)}$	$S_0^{2\,(b)}$	$K_0^{2\,(c)}$
^{10}B(s = 3) + ^{237}Np(S = 5/2)	64	1.167	86.39	2.644	137.90	194.88	141.24
^{10}B(s = 3) + ^{232}Th(S = 0)	60	1.126	73.12	2.4	125.33	177.15	136.29
^{11}B(s = 3/2) + ^{209}Bi(S = 9/2)	55	1.112	33.1	1.73	77.85	110.04	80.56
^{14}N(s = 1) + ^{209}Bi(S = 9/2)	78	1.136	43.8	1.98	90.10	127.23	95.11
^{12}C(s = 0) + ^{237}Np(S = 5/2)	72	1.107	81.14	2.55	134.41	189.88	224
^{12}C(s = 0) + ^{209}Bi(S = 9/2)	66	1.110	33.87	1.75	78.78	111.30	121.54

(a) Theoretical variances from SSM calculation LDP = $A/10$ and $E_{\text{d}} = 10$
(b) Theoretical variances from SSM calculation LDP = $A/20$ and $E_{\text{d}} = 10$
(c) Theoretical variances from best fit by using couple-channel spin effect

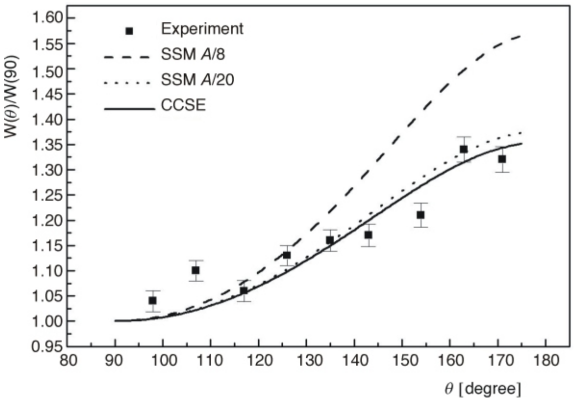

Figure 1. Experimental fission fragment angular distributions for ^{10}B + ^{237}Np. Dashed and dotted curves represent SSM calculations with LDP = $A/8$, and $A/20$, respectively. Solid curves represent the couple channel spin effect (CCSE)

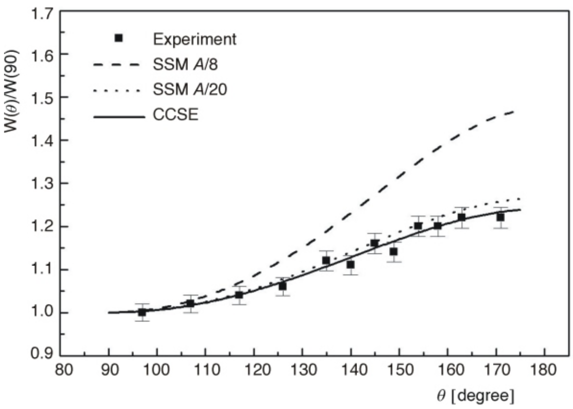

Figure 2. Experimental fission fragment angular distributions for ^{10}B + ^{232}Th. The dashed and dotted curves represent SSM calculation with LDP = $A/8$, and $A/20$, respectively. Solid curves represent the couple channel spin effect (CCSE)

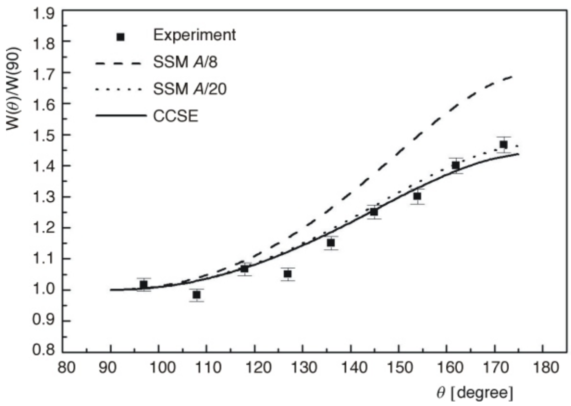

Figure 3. Experimental fission fragment angular distributions for ^{11}B + ^{209}Bi. The dashed and dotted curves represent SSM calculations with LDP = $A/8$, and $A/20$, respectively. Solid curves represent the couple channel spin effect (CCSE)

density of the nuclei in fission point (equation 7). The values obtained for by fitting the relation 3

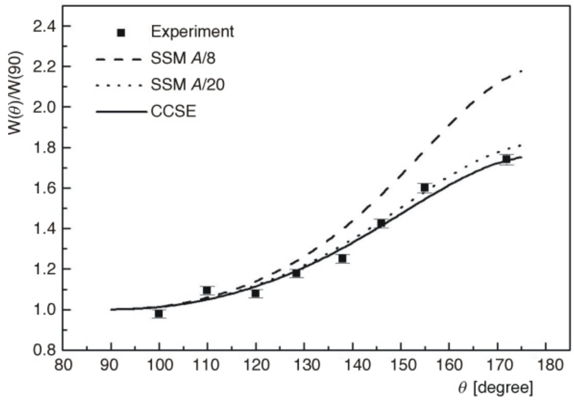

Figure 4. Experimental fission fragment angular distributions for ^{14}N + ^{209}Bi. The dashed and dotted curves represent the SSM calculation with LDP = $A/8$, and $A/20$, respectively. Solid curves represent the couple channel spin effect (CCSE)

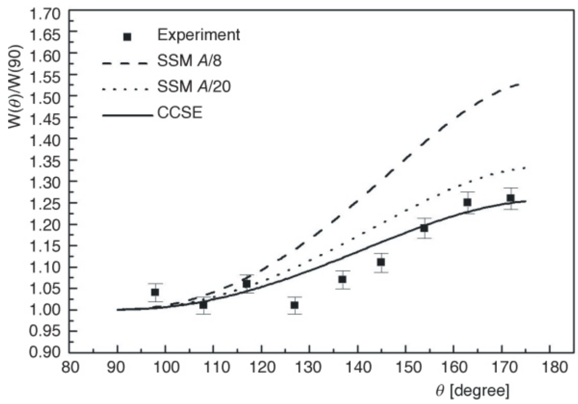

Figure 5. Experimental fission fragment angular distributions for ^{12}C + ^{237}Np. The dashed and dotted curves represent SSM calculations with LDP = $A/8$, and $A/20$, respectively. Solid curves represent the couple channel spin effect (CCSE)

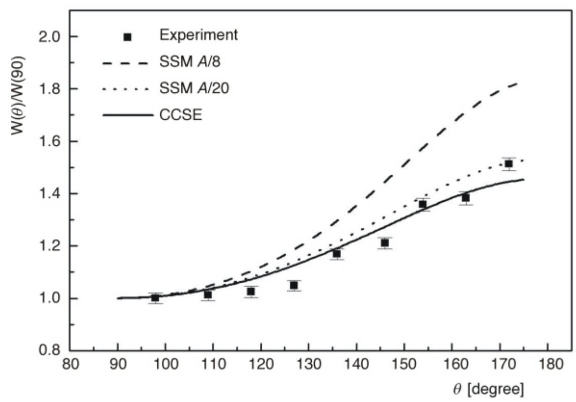

Figure 6. Experimental fission fragment angular distributions for ^{12}C + ^{209}Bi. The dashed and dotted curves represent SSM calculations with LDP = $A/8$, and $A/20$, respectively. Solid curves represent the couple channel spin effect (CCSE)

with experimental data are listed in the last column of tab. 1. Results show that in reactions in which the spin of the projectile is considerable, the variances of K_0^2 are around 20% smaller than the variances of S_0^2 in SSM, according to the LDP = $A/20$. But, in reactions such as $^{12}C + {}^{237}Np$, although the value of the target spin is considerable, there are no significant differences between K_0^2 and S_0^2. Therefore, couple channel spin formalism suggests that the effect of the projectile spin is more effective on angular anisotropy within the limits of energies near fusion barrier.

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УТИЦАЈ СПИНА НА АНИЗОТРОПИЈУ ФИСИОНИХ ФРАГМЕНАТА

Коришћењем модела статистичког реза извршена је анализа угаоне расподеле одабраног фисионог фрагмента, када је у индукованој фисији процењен бар један од спинова пројектила или мете. Резултати овог модела предвиђају да спинови пројектила или мете утичу на густину нуклеарног нивоа сложеног језгра. Експериментални подаци анализирани су формализмом упареног спинског ефекта. Овај формализам указује да спин пројектила више утиче на угаону анизотропију у енергетским границама блиским фузионој баријери.

Кључне речи: упарени спински ефект, фисионо-фузиона реакција, анизотропија угаоне расподеле