AN INVESTIGATION OF LOW-ENERGY PHOTON REFLECTION FROM THE IRON TARGET

by

Srpko MARKOVIĆ, Rodoljub SIMOVIĆ, and Vladan LJUBENOV

Received on October 24, 2003; accepted on May 5, 2004

For the photon transport kernel in form of the Thomson scattering function and a restrictive photon diffusion directed only toward free surface, several exact expressions of backscattered fluxes are demonstrated. The solving approach was established on a lemma proved by Placzek combined with the Fourier analytic inversion technique or the order of scattering method. Albedo problem in case of the homogeneous plane shield of iron subjected to the photons normally incident on the free surface is treated. Comparison of the results obtained by the analytical and Monte Carlo methods for the reflection of 40 and 60 keV photons from iron target confirms the domination of the single scattered photon flux and the strong influence of the scattering function anisotropy in reflection process at low energies.

Key words: low-energy photon reflection, Thomson scattering, Fourier transform method, Monte Carlo method, order of scattering method, iron shield

INTRODUCTION

It has been known that spreading of gamma radiation through ducts and voids within the radiation shield of nuclear facilities can be effectively examined by using the traditional albedo concept [1, 2]. This approach allows that radiation break-through can be described by multiple reflection from the radiation shield walls, demanding detailed knowledge of angular and energy distribution of the reflected particles. The same concept can be practically applied to calculate scattered radiation in the vicinity of a patient being exposed to X-ray diagnostic technique [3-5]. While in radiation protection of nuclear facilities we encounter high-energy radiation, in contrastive diagnostic techniques initial beams are below 100 keV. Low-energy range of gamma radiation enables one to approach the solving of the albedo problem not only by non-deterministic Monte Carlo methods, but also by analytic procedures with equal success.

Scientific paper UDC: 539.122...17:519.245 BIBLID: 1451-3994, *19* (2004), 1, pp. 39-45

VINČA Institute of Nuclear Sciences P. O. Box 522, 11001 Belgrade, Serbia and Montenegro

E-mail address of corresponding author srpkom@vin.bg.ac.yu (S. Marković)

In this paper some results obtained by applying the consistent analytical method for examining the radiation reflection, based on the previously developed technique are shown and analyzed [6-8]. When the energies of the initial radiation are close to the lower level of energy range for the X-ray medical diagnosis, such as few tens keV, photon scattering on a free electron can be presented by a Thomson cross section. By accepting this model of scattering, the basic transport problem is simplified, becoming the task equivalent to the energy independent transport. Further solving is based on the use of Placzek lemma, turning the problem of the radiation reflection from a flat wall into the equivalent transport problem in the infinite medium [9, 10], then on the application of Fourier transform on the transport equation, and on the analytical Fourier inversion of the specific solution.

As a result of the applied analytical procedure, angular density of the flux of reflected particles that diffuse through the shielding material constantly scattered in the boundary surface direction was calculated, and the obtained values were compared to the flux of the reflected particles once and twice scattered in the shield. Influence of anisotropy of photon scattering on reflection of low energy radiation was also analyzed and the effects of isotropisation of the scattering function were discussed. Results of the analytical calculations were compared to Monte Carlo simulation performed by MCNP-4C program [11]. Reflection of photons of the initial energies of 40 and 60 keV directed to the flat iron target at the angle of 90° was examined.

TRANSPORT EQUATION FOR LOW-ENERGY PHOTONS

The complex form of photon transport in which scattering and energy transfer simultaneously occur is usually described by the linear integrodifferential equation over the photon angular flux density $\phi(r, E, \Omega)$ [1]

$$\Omega \ \Delta \phi(r, E, \Omega) \ \mu(r, E) \phi(r, E, \Omega)$$
$$dE \ d\Omega \ \mu_s(r, E \ E, \Omega \ \Omega) \phi(r, E, \Omega)$$
$$S(r, E, \Omega)$$
(1)

Here, $\mu(r, E)$ represents the photon total interaction coefficient, $\mu_s(r, E = E, \Omega = \Omega)$ is the photon scattering kernel and $S(r, E, \Omega)$ is the photon source density.

For the low-energy photons in a homogeneous material characterized with an electron density N_e , the scattering kernel μ_s can be written as

$$\mu_{s}(r, E = E, \Omega = \Omega)$$

$$N_{e} \frac{r_{e}^{2}}{2} [1 \quad (\Omega = \Omega)^{2}] \delta(E = E) \qquad (2)$$

In eq. (2), as a low-energy limit of the Compton scattering, the Thomson cross section is used in which r_e stands for the classical electron radius.

If we expand the angular fraction of the scattering kernel μ_s in Legendre polynomials over the variable Ω Ω , eq. (1) becomes

$$\Omega \ \Delta \phi(r, E, \Omega) \quad \mu(E) \phi(r, E, \Omega)$$

$$N_e \frac{r_e^2}{2} \frac{2l}{l} \frac{2l}{0} \frac{1}{4\pi} \mu_{sl}^* \ \mathrm{d} \ \Omega \ P_l(\Omega \ \Omega) \phi(r, E, \Omega)$$

$$S(r, E, \Omega) \tag{3}$$

where the moments μ_{sl}^* are defined by

$$\mu_{sl}^{*} = 2\pi \frac{1}{4} d(\Omega \ \Omega) P_{l}(\Omega \ \Omega) [1 \ (\Omega \ \Omega)^{2}] (4)$$

The first three values of μ_{sl}^* are: $\mu_{s0}^* = 16\pi/3$, $\mu_{s1}^* = 0$ and $\mu_{s2}^* = 8\pi/15$, while the higher moments identically equal zero.

If we focus our investigation to a half-space of homogeneous material placed in the region x = 0 and exposed to a monodirectional and monoenergetic circularly symmetric current of photons injected from the vacuum, the photon transport equation gets a new, simpler form

$$\omega \frac{\partial \phi(x, E, \omega)}{\partial x} \quad \mu(E) \phi(x, E, \omega) \quad \frac{8\pi}{3} N_e r_e^2$$

$$\sum_{l=0}^{2} \frac{2l}{2} \frac{1}{2} \mu_{sl} P_l(\omega) \prod_{l=1}^{1} P_l(\omega) \phi(x, E, \omega) d\omega \quad (5)$$

with the boundary condition

$$\phi(0, E, \omega) \quad \frac{1}{\omega_0} \delta(\omega \quad \omega_0) \delta(E \quad E_0), \omega_0 \quad 0 \ (6)$$

Here, we have defined the new moments $\mu_{sl} \quad \mu_{sl}^* \mid \mu_{s0}^*(\mu_{s0} \quad 1, \mu_{s1} \quad 0, \mu_{s2} \quad 1/10)$ and sli g-htly modified denotation ($\omega \quad \Omega_x$).

In deriving eq. (5) we have utilized a symmetry property of the photon angular flux density, *i. e.*, the fact that in plane geometry with the proposed boundary condition (6), identity $\phi(r, E, \Omega) = \phi(x, E, \omega)$ is valid. Moreover, in formulating eq. (5), the addition theorem of Legendre polynomial [12] has also been employed.

Applying the scaling transformation of the spatial variable $t = \mu(E)x$ the transport eq. (5) can be written in a more familiar form

$$\omega \frac{\partial \phi(t, E, \omega)}{\partial t} \quad \phi(t, E, \omega)$$

$$c(E) \int_{l=0}^{2} \frac{2l-1}{2} \mu_{sl} P_{l}(\omega) \int_{1}^{1} P_{l}(\omega) \phi(t, E, \omega) d\omega \quad (7)$$

where

$$c(E) \quad \frac{8\pi}{3} \frac{N_e r_e^2}{\mu(E)} \tag{8}$$

The basic characteristic of eq. (7) is that it describes a pseudoenergy transport problem, *i. e.*, in this case the dependency of the photon angular flux density on the variable of E is of parametric shape. Such an equation is solved as an one-velocity transport equation with anisotropic scattering function [13]. The solution characteristics depend a great deal on the c(E) function – the equivalent multiplication parameter of the transport problem.



Figure 1. Equivalent multiplication parameter c(E)

The c(E) function for iron and lead within the photon energy range of 20-80 keV is shown in fig. 1. It is evident that in the energy range in question values of the c(E) parameter are very low, especially for lead, which points to reflection where the component of single collided photons is dominant. With the increase of the initial radiation energy the values of multiplication parameter increase as well, thus turning the reflection process into a more complex transport phenomenon connected to multiple collisions of the initial photons before definite backscattering.

Transport equation of the scattered photons

As the unscattered part of photons does not participate in backscattered radiation, it is practical to separate this portion of the flux from the whole beam of photons

$$\phi(t, E, \omega) \quad \phi^{0}(t, E, \omega) \quad \phi^{s}(t, E, \omega) \quad (9)$$

and resolve an appropriate equation for the scattered photons $\phi^s(t, E, \omega)$

$$\omega \frac{\partial \phi^{s}(t, E, \omega)}{\partial t} \quad \phi^{s}(t, E, \omega)$$

$$c(E)_{l=0}^{2} \frac{2l}{2} \frac{1}{2} \mu_{sl} P_{l}(\omega)_{1}^{1} P_{l}(\omega) \phi^{s}(t, E, \omega) d\omega$$

$$c(E)_{l=0}^{2} \frac{2l}{2} \frac{1}{2} \mu_{sl} P_{l}(\omega) P_{l}(\omega_{0}) \frac{e^{-t/\omega_{0}}}{\omega_{0}} \delta(E - E_{0}) (10)$$

with the boundary condition $\phi^s(0, E, \omega) = 0$ for $\omega = 0$.

The exact solution of eq. (10) cannot be achieved in the form of an analytic function or even as a compound combination of the analytic functions [7, 14]. Namely, if the analytically exact solution of a specific transport equation exists, it always includes one or more exponential singular integrals. Therefore, we would prefer to resolve a somewhat modified transport equation corresponding to the photons with specific diffusion histories, for which the exact solution exists as a combination of simple analytic functions.

ANALYTICAL DETERMINATION OF REFLECTED PHOTONS

If we limit our goal to defining only the angular flux density of photons $\phi^a(t, E, \omega)$ which strictly move into directions ω 0 after each successive collision until the last one, then we have to resolve the adequate transport equation similar to the previous one, except that the upper limit of the integral on the right hand side of eq. (10) now equals zero. The solving procedure is based on the application of the Fourier transform method to the corresponding transport equation over the space coordinate t, as well as on using the Placzek lemma for converting a half-space transport problem into an equivalent infinite medium problem. These techniques lead us to the integral equation in the Fourier transform space

$$(1 \quad ik\omega)F^{a}(k, E, \omega)$$

$$c(E)^{2}_{l\ 0}\frac{2l}{2}\frac{1}{2}\mu_{sl}P_{l}(\omega)^{0}_{1}F^{a}(k, E, \omega)P_{l}(\omega)d\omega$$

$$c(E)^{2}_{l\ 0}\frac{2l}{2}\frac{1}{2}\mu_{sl}P_{l}(\omega)\frac{P_{l}(\omega_{0})}{1\quad ik\omega_{0}}\delta(E \quad E_{0})$$

$$\omega\phi^{a}(0 \quad E, \omega) \qquad (11)$$

where $F^{a}(k, E, \omega)$ means the Fourier transform of the photon angular flux density $\phi^{a}(t, E, \omega)$

$$F^{a}(k, E, \omega) = \int_{-\infty}^{\infty} e^{-ikt} \phi^{a}(t, E, \omega) dt$$
 (12)

and k is the complex variable.

The analytic expression for the backscattered photons $\phi^a(0, E, \omega)$ can be readily obtained from eq. (11) by applying the same mathematical procedure previously developed in the study of the similar problem of energy independent particle transport [7]

$$\phi^a(0, E, \omega) = \frac{c(E)}{\omega \omega_0}$$

$${}^{2}_{l=0}\frac{2l-1}{2}\mu_{sl}P_{l}(\omega)\frac{Q_{l}(\omega_{0})}{D(\omega_{0})}\delta(E-E_{0}), \ \omega = 0 \quad (13)$$

where the functions D^- and Q_l are defined by

$$D(u) = 1 \frac{c(E)}{2} [A_{00}(u) \quad 5\mu_{s2}A_{22}(u)] + 5\frac{c^{2}(E)}{4} \mu_{s2} [A_{00}(u)A_{22}(u) \quad A_{02}^{2}(u)]$$
(14)

and

$$Q_0(u) = 1 - \frac{5}{2}c(E)\mu_{s2}[A_{02}(u)P_2(u) - A_{22}(u)](15)$$

$$Q_2(u) \quad P_2(u) \quad \frac{c(E)}{2} [A_{02}(u) \quad A_{00}(u)P_2(u)] (16)$$

Additionally, the functions $A_{nl}(u)$ are

$$A_{00}(u) \quad u \ln 1 \quad \frac{1}{u}$$
 (17)

$$A_{02}(u) = \frac{1}{4} 6u^2$$
 3*u* 2*u* ln 1 $\frac{1}{u}$

$$6u^3 \ln 1 \frac{1}{u}$$
 (18)

and

$$A_{22}(u) \quad \frac{1}{16} \ 36u^4 \ 18u^3 \ 12u^2 \ 3u$$

$$24u^{3}\ln 1 \quad \frac{1}{u} \quad 4u\ln 1 \quad \frac{1}{u} \quad 36u^{5}\ln 1 \quad \frac{1}{u} \quad (19)$$

Solutions by the order of scattering method

Applying the order of scattering method [15] to eq. (10), it is easy to derive the first order solution – the angular flux density of reflected photons scattered only once

$$\phi^{1}(0, E, \omega) \quad \frac{c(E)}{2} \frac{1}{\omega_{0} \quad \omega}$$

$$\sum_{l=0}^{2} (2l \quad 1) \mu_{sl} P_{l}(\omega) P_{l}(\omega_{0}) \,\delta(E \quad E_{0}), \quad \omega \quad 0 \quad (20)$$

and, in the case of isotropic scattering function, the second order solution – the angular flux density of photons backscattered twice

$$\phi^{2}(0, E, \omega) \quad \frac{c^{2}(E)}{4} \frac{1}{\omega_{0} \quad \omega} \quad \omega_{0} \ln 1 \quad \frac{1}{\omega_{0}}$$
$$\omega \ln 1 \quad \frac{1}{\omega} \quad \delta(E \quad E_{0}), \quad \omega \quad 0 \qquad (21)$$

In principle, the higher terms of iterative series (ϕ^3 , ϕ^4 ,...) can be obtained consecutively one by one, but in practice the mathematical expressions become too large.

Figure 2 shows the angular dependence of the backscattered fluxes $\phi^a(0, E, \omega)$ and $\phi^1(0, E, \omega)$ for a few different values of the parameter c(E) and for a unit intensity monodirectional current of photons injected normally $(\omega_0 \quad 1)$ into the boundary surface. The selected plane geometry and the normal angle of photon incidence correspond quit well to the actual conditions of the medical examinations performed by the contrast X-ray diagnostic technique [3, 4]. One can see from fig. 2 that the values of $\phi^a(0, E, \omega)$ are always higher than the values of $\phi^{1}(0, E, \omega)$. This result confirms that the flux $\phi^{a}(0, E, \omega)$ describes a radiation diffusion mechanism with more complex photon histories involved than is the case with the flux $\phi^{1}(0, E, \omega)$, which represents only the once backscattered photons.



Figure 2. The angular dependence of backsattered photons for different values of the parameter *c* and the angle of incidence deffined by $\omega_0 = 1$



COMPARISON OF THE ANALYTICAL SOLUTIONS AND MONTE CARLO SIMULATION FOR THE IRON TARGET

The first and the second order solution of the transport eq. (10), *i. e.*, eqs. (20) and (21), are applied in order to approximate analytically the low-energy photon reflection from the plane iron shield. However, the photon reflection is usually determined in full by Monte Carlo simulation [11, 16, 17]. Here, the MCNP-4C code is used with the standard photon nuclear data library MCPLIB [11]. The data for the reflected photons are collected in 10 angular and 20 energy intervals for 40 keV as well as 60 keV perpendicularly injected photons. Simulations have included 10⁹ photon histories resulting in statistical uncertainty of less than 1% for each angular-energy domain.

The analytical and Monte Carlo results for the differential albedo, defined as the ratio of energy integrated backscattered angular flux and the current of initial photons, are shown in fig. 3. The Monte Carlo

values are presented by open circles, while analytical results are shown by dotted and full lines for the first order solution and the sum of the first and second order solutions, respectively. It is evident that the single scattering flux with the Thomson kernel as a photon scattering function approximates correctly the half-space reflection in the case of 40 keV initial photons. For the case of 60 keV incident photons, a Monte Carlo result emphasizes the less symmetrical angular behavior of the reflected photons. The analytical results obtained as a sum of the once and twice backscattered photons differ by about 10% from the Monte Carlo results. However, better agreement is in the middle of the angular interval, while at both ends of the interval the analytical approximation is less satisfactory. It is the consequence of the incapability of the Thompson function to describe accurately the process of Compton scattering peaked in the forward direction at higher energies.

The angular dependence of the backscattered flux $\phi^1(0, E, \omega)$ calculated from eq. (20) for two forms of Thomson scattering function and for 40 keV initial



Figure 4. Angular distribution of reflected photon flux for 40 keV photons perpendicularly injected into the plane iron shield and for two forms of Thomson scattering function



Figure 5. Angle integrated albedo coefficients for the initial photons of 40 keV injected normally into the plane iron target

photons is shown in fig. 4. The anisotropic case is represented by the full line while the isotropic kernel approximation (μ_{s1} μ_{s2} 0) is given by the dotted line. The Monte Carlo results are given by the open circles. It is evident that the single scattering flux with the Thomson kernel in anisotropic form approximates correctly total half-space reflection in the low-energy domain of initial photons. We can conclude that the better agreement could be achieved by including the model of photon multiple scattering represented by the expression (13). On the contrary to these results, the single scattering model with the Thomson kernel in isotropic form does not follow the global shape of the angular reflected flux.

Figure 5 shows the angle integrated albedo coefficients for the three chosen angular intervals (dotted lines) and for the whole interval of backscattered angles (full line). The initial photon energy is again of 40 keV and the average energy of the total reflected photon current is determined as \overline{E}_0 35.35 keV. The pick shape forms of the integrated albedo coefficients over the chosen angular intervals are evident. Moreover, for each of these intervals the average energy of reflected photon current fully corresponds to the angle-energy transfer law. These results also approve the adequacy of the single collision model in photon low-energy reflection.

CONCLUSION

Good agreement of results for the reflected photon flux and the differential albedo, obtained by the analytical approach and the Monte Carlo simulation for 40 and 60 keV photons penetrating perpendicularly into the iron target, confirms, that in the low-energy domain photon reflection can efficiently be described by the first and second order solution of the appropriate transport equation. The Thomson scattering function in anisotropic form, which suites quite well the applied analytical technique, is also reliable in this energy range. Based on the exact solutions, especially on the expression (13) for the angular flux density of photons which strictly move into directions $\omega = 0$ after each consecutive collision, the half-space buildup factor could be formulated entirely in an analytic form.

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Српко МАРКОВИЋ, Родољуб СИМОВИЋ, Владан ЉУБЕНОВ

ПРОУЧАВАЊЕ РЕФЛЕКСИЈЕ НИСКОЕНЕРГЕТСКИХ ФОТОНА ОД ГВОЗДЕНЕ МЕТЕ

У раду је изведено више аналитичких израза за флукс рефлектованих фотона под претпоставкама да је функција расејања фотона у виду Томсоновог пресека и да се фотони по продирању у мету рестриктивно простиру искључиво према слободној површини. Формуле су добијене решавањем транспортне једначине фотона применом Плачекове леме повезане са инверзном Фуријеовом трансформацијом, или са методом судар по судар. Разматран је албедо проблем фотона који под правим углом продиру у хомогени равански штит од гвожђа. Поређење резултата добијених аналитичким и Монте Карло методама за иницијалне енергије фотона од 40 и 60 keV, потврђује да при нискоенергетској рефлексији доминира процес једном уназад расејаних фотона са јаким утицајем анизотропије функције расејања.