THREE-DIMENSIONAL MODEL OF TRACK GROWTH: COMPARISON WITH OTHER MODELS

by

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Here, we present a three-dimensional model of track growth in nuclear track detectors. The equation for the track wall in three dimensions and the equation of the contour line of the track opening have been derived for all types of tracks (*i. e.*, tracks with sharp tips and tracks with rounded tips). The expression for the surface area of the track opening has also been found. The equations become the well-known expressions for minor and major axes for the special case of constant track etch rates.
Computations of track parameters based on our model have been compared with the track growth models given by Somogyi and Szalay and the one given by Fews and Henshaw. Good agreements have been found among these three independent models.

Key words: solid state nuclear track detector, track growth, model

INTRODUCTION

The geometry of track development has attracted much attention for a long time (*e.g.*, Henke and Benton, [1], Paretzke *et al.* [2], Somogyi and Szalay [3], Somogyi [4], Fromm *et al.* [5], Hatzialekou *et al.* [6], Ditlov [7], Meyer *et al.* [8], Nikezić and Kostić [9]). During the etching, a track passes through different phases. At the beginning, the track tip is sharp. With the prolonged etching when the etching passes the ending point of a particle trajectory, the track tip becomes rounded. The track opening also passes through different phases and it was analyzed in details by Somogyi and Szalay [3]. The track opening may be circular in shape when the incident angle is 90 ; if the angle is oblique, the track opening is elliptical, or in the shape of an ellipse + circle, or circular, depending on the etching, energy and angle (for track etch rate V_t = const.). If track etch rate is V_t const., the track opening is semi-elliptical or even a more complex "egg-like" geometrical figure. The models of the track growth presented by other authors are mostly two-dimensional. Some elements of three-dimensionality may be found in Somogyi and Szalay [3] and Fews and Henshow [10], but without rigorous treatment.

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THE EQUATION OF THE TRACK WALL IN TWO DIMENSIONS

The equation of the track wall can be derived in the following way. Referring to fig. 1, the point A on the track wall with coordinates (x, y) was formed from the point x_0 on the particle track. From the point (0, 0), the etching travels with the track etch rate V_t along the x-axis (which is the particle trajectory) and reaches the point x_0 at the time t_0. From x_0, the etching progresses to the point A with the bulk etch rate V_b. The angle $\delta = \delta(x_0)$ is the angle between V_t and V_b at the point x_0, as shown in fig. 1, and can be found as

$$\delta(x_0) \quad a\sin\frac{1}{V(x_0)} \tag{1}$$

where

$$V \quad \frac{V_t(x_0)}{V_b}$$

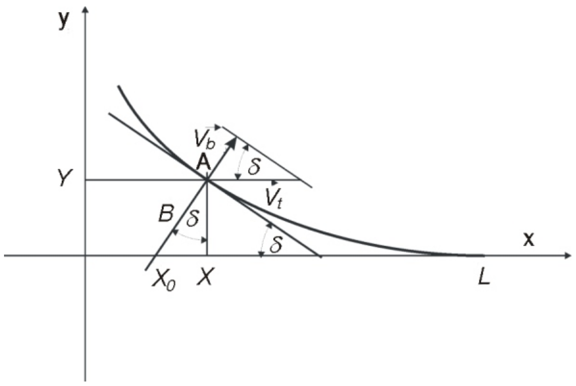

Figure 1. Derivation of a two-dimensional equation of the track wall

From the geometrical point of view, it is clear that

$$y(x) \quad \text{tg}\,\delta(x_0) \quad \frac{1}{\sqrt{V^2(x_0)\ \ 1}} \tag{2}$$

This equation cannot be used as the track wall equation because the expression on the right depends on x_0, while the expression on the left depends on x. Noting that $x_0 = x - \Delta x$, we have

$$\Delta x \quad y(x)\ \text{tg}\,\delta(x_0) \quad y(x)\ y(x) \tag{3}$$

so we can obtain

$$x_0 \quad x \quad y(x)\ y(x) \tag{4}$$

and

$$y \quad \frac{1}{\sqrt{V^2(x \quad yy)\ \ 1}} \tag{5}$$

This is the equation of the track wall in the differential form with both sides depending only on x. Unfortunately, this equation cannot be solved analytically. If the angle δ is small or if $V(x)$ is a slowly varying function (which is usually the case in many applications) (see for example Dorschel *et al* [11, 12], $y\,y$ in the denominator of eq. (5) can be neglected and the *aproximation equation* of the track wall becomes

$$y \quad \int_x^L \frac{dx}{\sqrt{V^2(x)\ \ 1}} \tag{6}$$

This approximation equation was previously used by Nikezić [11] for the analytical three-dimensional determination of the track parameters.

The coordinates (x, y) of the point A can be calculated in a simpler way, *i. e.*,

$$y \quad B\cos\delta(x_0) \tag{7a}$$

and

$$x \quad x_0 \quad B\sin\delta(x_0) \tag{7b}$$

where

$$B \quad V_b(T \quad t_0) \tag{7c}$$

and T is the total etching time. By using eqs. (7a) to (7c), the coordinates of the points on the track wall can be generated. The best fit will give

$$y \quad F(x, L) \tag{7d}$$

as the equation of the wall, where L is the distance penetrated by the etching solution (see fig.1). However, the information about $V_t(x)$ is lost in this way. Another possibility is to solve eq. (5) numerically, but this may be more complicated.

In the special case where $V_t/V_b = V =$ const., the track wall is represented by a line in two dimensions, and eq. (7) becomes

$$y_{\text{linear}} \quad F(x, L) \quad \frac{x \quad L}{\sqrt{V^2 \ \ 1}} \tag{7e}$$

The equation of the track wall in the conical phase in three dimensions for normal incidence can be written as

$$\sqrt{x^2 \quad y^2} \quad F(z, L) \tag{8}$$

where the z-axis is along the particle trajectory, and (x, y) are coordinates of the points in the track wall. The track opening is circular in shape when incidence is normal, but in an egg-like shape or a droplet-like shape when the incidence is oblique. The contour equation for the opening is given by

$$\sqrt{x^{\ 2} \quad y^{\ 2}\sin^2\theta} \quad F(y \ \cos\theta \quad \frac{h}{\sin\theta}, L) \tag{9}$$

where x and y are coordinates on the contour of the track opening, θ is the incident angle with respect to the detector surface and h is the total removed layer.

In the cases where V is not a constant, the track opening is not an ellipse, but is egg-like instead, or has even more complicated shapes depending on the functions F and ultimately on the function V.

OVER-ETCHED TRACKS: NORMAL INCIDENCE

In this section, the over-etched tracks will be considered. The schematic sketch of an over-etched track in two dimensions is shown in fig. 2. After a certain time the etching will reach the end point E of the particle range. At that time, the wall of the track is formed and denoted by the number 1 in fig. 2. The point A with coordinates (z, y) is contained in the track wall. The angle between the tangent t on the wall at the point A and the z-axis is the local development angle δ. Although further etching will progress in all directions with the same rate V_b, including the surroundings of the point E, it can be characterized by two processes for easier treatment, namely, the displacement for the distance d of the wall from the position 1 to the position 2, and the formation of a sphere around the point E with same diameter d. The point A moves normally onto the tangent t for a distance d to the point A' with coordinates $(z \; , y \;)$. The track wall consists of two parts, *i. e.*, semi-conical and spherical. These two parts intersect at points M_1 and M_2 (if the problem is considered in two dimensions). In the three-dimensional representation, intersection occurs at a circle, which is schematically shown in fig. 2.

From the geometrical point of view, the relationships between the (z, y) and $(z \quad y \;)$ coordinates are given by

$$z \quad z \quad d \sin\delta \text{ and } y = y + d \cos\delta \qquad (10)$$

By using eq. (7d) with $L = R$, and combining with eq. (10), one can obtain the equation for the semi-conical part (in the over-etched phase) in two dimensions as:

$$y \quad F(z, R) \quad d \cos\delta \qquad (11)$$

or

$$y \quad F(z \quad d \sin\delta, R) \quad d \cos\delta \qquad (12)$$

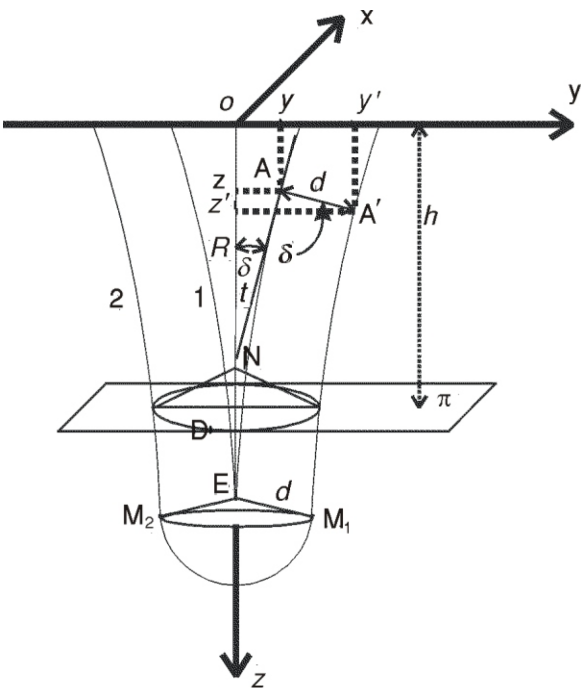

Figure 2. Schematic sketch of an over-etched track in three dimensions

Now, we can formally replace y with y, and z with z, to get the equation for the track wall in two dimensions as

$$y \quad F(z \quad d \sin\delta, R) \quad d \cos\delta \qquad (13)$$

This is the equation of the track wall in two dimensions at the moment when the etching reaches the end point of the particle trajectory.

The equation of the track wall in three dimensions is given analogously to eq. (8) as

$$\sqrt{x^2 \quad y^2} \quad F(z \quad d \sin\delta, R) \quad d \cos\delta \qquad (14)$$

The detector surface after etching is represented by the plane in fig. 2. The plane is normal to the z-axis which represents the particle trajectory. The thickness of the removed layer is denoted by h (see fig. 2). If $z = h$ is substituted into eq. (14), the equation of a circle in the plane is obtained as

$$\sqrt{x^2 \quad y^2} \quad F(h \quad d \sin\delta, R) \quad d \cos\delta \quad D \qquad (15)$$

where D is the radius of the circle and (x, y) are the coordinates of points on the circle. The angle δ in eqs. (14) and (15) is the local developing angle at the point N (fig. 2). All points of the track-opening contour were developed from the same point N on the particle path under the same developing angle δ. This remark can be generalized to all points with the same z and same developing angle δ. The diameter, $D = 2\,D$, of the circular track opening is found as

$$D \quad 2D \quad 2[F(h \quad d \sin\delta, R)] \quad d \cos\delta \qquad (16)$$

OBLIQUE INCIDENCE

In this part, the case of the oblique incidence is considered. Apparently, this situation is more complicated than the previous one. However, if two transformations of the coordinate system are applied, the problem will be simplified significantly.

Semi-elliptical opening

Semi-elliptical openings are found when the detector surface after etching has not crossed the part of the sphere formed around the point E. In this section, such kind of tracks will be considered. The geometry used for considering a semi-elliptical track opening is presented in fig. 3. The nomenclature is the same as that in the previous figures. A new parameter is the angle θ which is the incident angle with respect to the detector surface. Around the end point E of the particle trajectory, a sphere with radius d is formed which is joined with the semi-conical part of the track wall. The plane $_1$ represents the detector surface after etching, and h is the thickness of the removed layer. The track is "cut" by the plane $_1$ under the angle θ with respect to the particle direction (z-axis). The first step is a translation of the coordinate system (x, y, z) from the point O to the point O with coordinates O (0, 0, z_0) where $z_0 = = h/\sin\theta$. The newly obtained system (x y , z) is related to the original one through the equations

$$x \quad x, \quad y \quad y \text{ and } z \quad z \quad z_0 \qquad (17)$$

Equation (14) for the track wall in the new coordinate system becomes

$$\sqrt{x^2 \quad y^2} \quad F(z \quad z_0 \quad d\sin\delta, R) \quad d\cos\delta \qquad (18)$$

The second step is a rotation of the (x ,y ,z) coordinate system through an angle (/2 – θ) around the x -axis. The newly formed coordinate system (x , y , z) is related to the (x , y , z) system through the equations

$$y \quad y \sin\theta \quad z \cos\theta$$

and (19)

$$z \quad y \cos\theta \quad z \sin\theta$$

Equation (18) for the track wall in the (x , y , z system is now

$$\sqrt{x^2 \quad (y \sin\theta \quad z \cos\theta)^2}$$

$$F(z \sin\theta \quad y \cos\theta \quad z_0 \quad d\sin\delta, R) \quad d\cos\delta \qquad (20)$$

The surface of the detector after etching is given as $z = 0$. By substituting $z = 0$ into eq. (20), the intersection between the track wall and the new detector surface described by $z = 0$ is given as

$$\sqrt{x^2 \quad y^2 \sin^2\theta^2}$$

$$F(y \cos\theta \quad z_0 \quad d\sin\delta, R) \quad d\cos\delta \qquad (21)$$

Here, x and y are the coordinate axis along the plane $_1$ (both belonging to the plane $_1$) and z is normal to $_1$. In the case, y is extended along the major axis of the track and x is normal to it. This is the equation for the contour line of the track opening in the semi-elliptical phase, where the track is rounded but has not yet passed the spherical shape. The angle δ, which appears in the eq. (21) implicitly and varies along the contour line, makes calculations difficult. However, the calculation of the contour line is facilitated by the fact that all points with the same value of z have the same developing angle δ (as emphasized before).

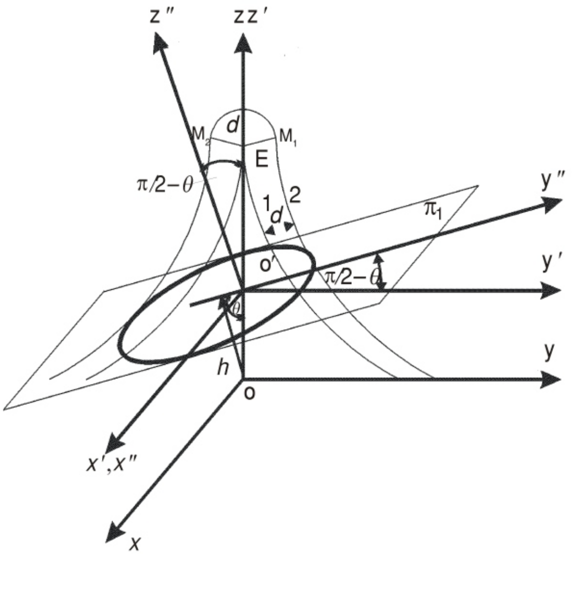

Figure 3. Oblique incidence

Some consequences of eq. (21)

Track length (major axis)

The track length can be found from eq. (21) when $x = 0$. Here, the coordinates y_1 and y_2 where the contour line crosses the y -axis are found as

$$y_{1,2} \sin\theta \quad [F(y_{1,2} \cos\theta$$

$$z_0 \quad d\sin\delta, R) \quad d\cos\delta] \qquad (22)$$

Note that unknown variables y_1 and y_2 are on both sides of eq. (22) and iterations are needed to solve the equation. The length D of the track opening is then equal to

$$D \quad |y_1| \quad |y_2| \qquad (23)$$

Track width (minor axis)

The track width cannot be found by taking $y = 0$ because the center of the opening is shifted along the y - axis. In this case, the maximum of the function given in eq. (21) should be determined by locating

$$\frac{dx}{dy}_{y_{max}} \quad 0 \qquad (24)$$

where y_{max} is the value of y when x has a maximum. Then y_{max} should be substituted into eq. (21) to find the maximum value x_{max}. The track width (minor axis of the track opening) is given by $d = 2x_{max}$. Such procedures are rather complicated and impractical because the angle δ also depends on the coordinate y. A better approach is to perform calculations of x from eq. (21) and to determine the maximal value of x by systematically changing the values of y from y_1 to y_2.

Surface area of track opening

The surface area S of the track opening can also be found from eq. (21) by performing the integration

$$S \quad 2 \int_{y_1}^{y_2} \sqrt{F^2(y \cos\theta \quad z_0 \quad d \sin\theta \quad d \cos\delta) \quad y^2 \sin^2\theta dy} \tag{25}$$

where y_1 and y_2 are determined by eq. (22). Numerical integration is needed to determine S.

Track opening in transitional phase

The plane representing the detector surface after etching intersects part of the sphere formed around the point E. Consequently, the track opening consists of two parts, namely, semi-elliptical and circular. The geometry, although similar to the previous case, is presented separately in fig. 4. The two parts of the track wall, semi-conical and spherical, are joined to the circle at the points M_1 and M_2. The plane $_2$ corresponding to the detector surface will cut the track after etching, so both the semi-conical and the spherical parts will be crossed. As a result, the complicated curve, namely, circle + semi-ellipse, is formed in the $_2$ plane, which is also presented in fig. 4 in bold. The circle and the semi-ellipse are also joined at the points denoted by A and A in fig. 4. The semi-elliptical part of the track opening (lower left direction from the line A A) is represented by the same eq. (21) as was derived above. The circular part is represented by the equation of a circle, and the parameters to be determined are only the radius of the circle and the coordinates of the center in the $_2$ plane. The equation of the sphere in the (x, y, z) system with the center at the point with coordinates (0, 0, R) and radius d is

$$x^2 \quad y^2 \quad (z \quad R)^2 \quad d^2 \tag{26}$$

Now, the procedures for translation of the coordinate system to the point (0, 0, z_0) and the rotation through an angle (/2 – θ) around x should be repeated. After these transformations see eqs. (17 and 19), the equation of the sphere in the (x″, y , z sys-tem becomes

$$x^{\ 2} \quad (y \ \sin\theta \quad z \ \cos\theta)^2$$

$$(y \ \cos\theta \quad z \ \sin\theta \quad z_0 \quad R)^2 \quad d^2 \tag{27}$$

The intersection with the plane $_2$, which has the equation z = 0 in the (x , y , z) coordinate system, gives the equation of the circle as

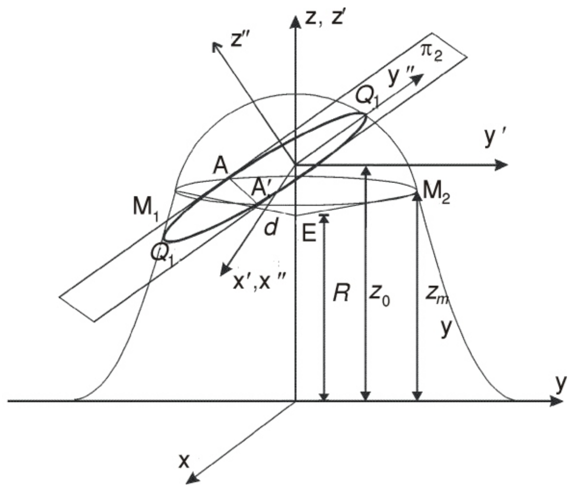

Figure 4. Transitional phase of the track development

$$x^{\ 2} \quad [y \quad \cos\theta(z_0 \quad R)]^2$$

$$d^2 \quad (z_0 \quad R)^2 \sin^2\theta \tag{27'}$$

Therefore, the center and the radius of the circle can be found from this equation, viz., the center is at the point $(0, -(z_0 - R)\cos\theta)$ in the $_2$ plane. The radius r of the circle is equal to

$$r \quad \sqrt{d^2 \quad (z_0 \quad R)^2 \sin^2\theta} \tag{28}$$

Major and minor axes

The minor axis d is found through the largest value of the x coordinate, x_{max}, of the contour line, regardless of whether it belongs to the circular or the semi-elliptical part of the opening

$$d \quad 2x_{max} \tag{29}$$

For determination of the major axis, the three-dimensional approach is not needed. The major axis is equal to the distance Q_1Q_2, fig. 5. The coordinates of Q_2 are found from eq. (27) by taking $x = 0$ while Q_1 is the same as y_1 in eq. (22).

Surface area of the track opening

The surface area is equal to the sum of those for the circular and semi-elliptical parts.

Track opening in circular phase

As etching progresses, the circular part will constitute a larger proportion of the track opening. Ultimately, the entire opening will become circular and the track is completely spherical. The major and minor axes of the track opening are then equal to the

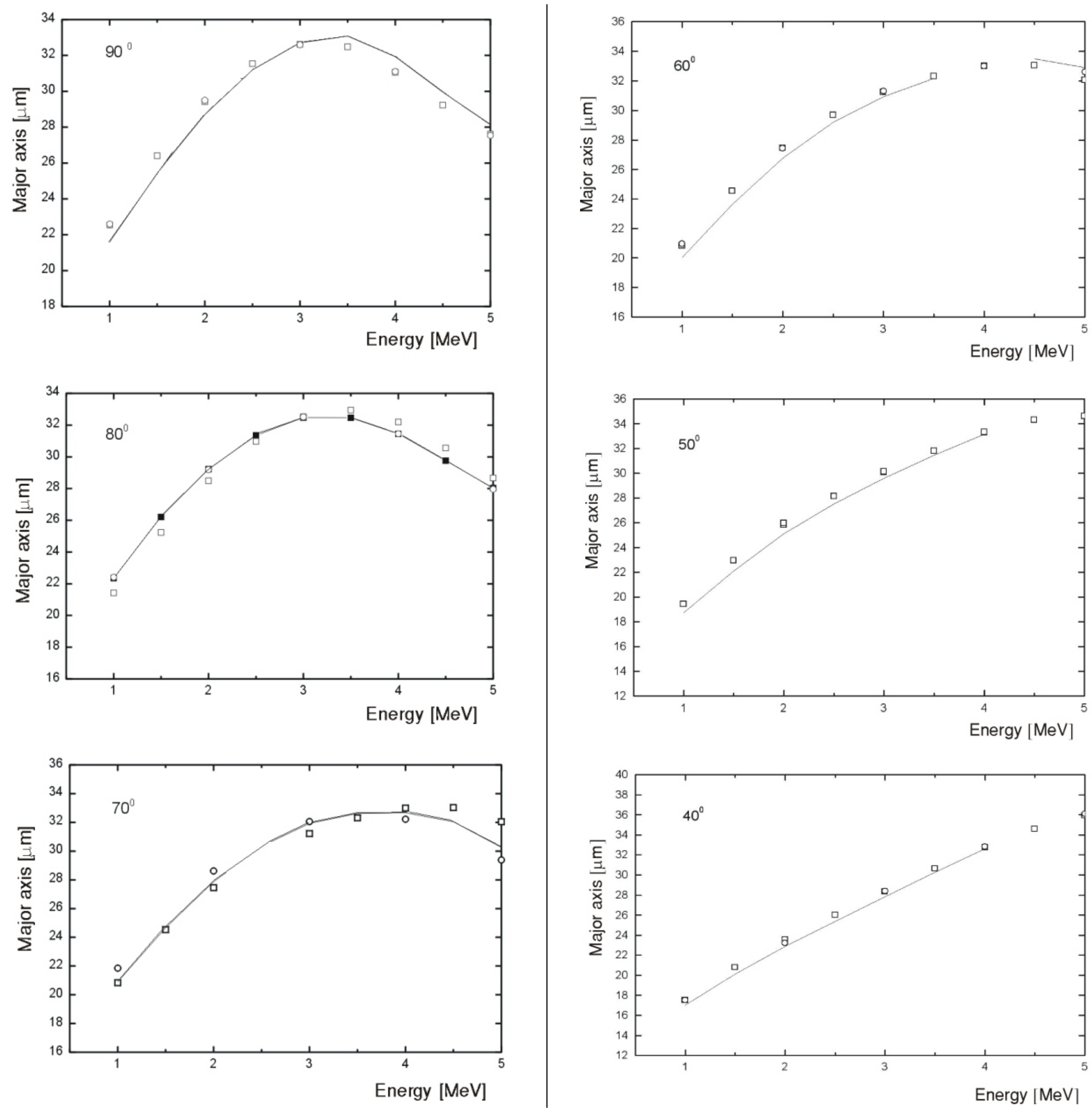

Figure 5. Major axis calculated by three different models of the track growth; solid line – Somogyi and Szalay model, open circles – Fews and Henshaw model, open squares – Nikezić and Yu model

diameter of the circle, which can also be found from eq. (28).

CALCULATION OF THE TRACK PARAMETERS AND COMPARISON WITH OTHER MODELS

Our model of the track growth presented above was employed to calculate the major-axis length D and the minor-axis length d of alpha-particle tracks in the CR-39 detector. The input parameters in the model include the incident energy and angle of the alpha particle, etching time, the bulk-etch rate V_b and the V_t function, *i. e.*, the etching rate along the particle track. The ratio $V(R') = V_t/V_b$ can be used instead, where R' is the residual range of the particles. It has been shown earlier that the obtained values for D and d vary significantly with the V_t function.

We have used V functions found in the literature for alpha particles in the CR-39 detector referred to as the Green's function in the present work [11]

$$V = 1 + (a_{1G} e^{-a_{2G} R'} + a_{3G} e^{-a_{4G} R'})(1 - e^{-a_{5G} R'})$$

with

$$a_{1G} = 11.45, \quad a_{2G} = 0.339, \quad a_{3G} = 4, \quad a_{4G} = 0.044, \quad \text{and} \quad a_{5G} = 0.58 \qquad (30)$$

Some authors have used the V function in the form of $V = a(R')^{-b}$ where a and b are constants. Such functions are not considered in the present work because V will become too large when R' is very small; also, these functions have no maxima and do not represent the realistic situation. In addition, our software requires $V = 1$ at $R' = 0$.

We have performed calculations for the lengths of major and minor axes of the track openings through the functions given in eq. (30) above. Another set of calculations has been performed with the method given by Somogyi and Szalay [3], also with the function given in eq. (30), and the third group of calculations has been done with the method described by Fews an Henshaw [10]. Results are given in fig. 5.

In all calculations the range of alpha particles in CR-39 detector was determined by SRIM2000 code [12].

Comparison between Somogyi-Szalay, Fews-Henshaw, and Nikezić-Yu models

The lengths for the major axes were calculated for the incident energies 1 MeV to 5 MeV (with the step of 0.5 MeV) and angles 40^0 to 90^0 (with the step 10^0) of alpha particles and the results are given in fig. 5. All calculations were done with Green's V_t function. Solid lines are obtained with the model of Somogyi and Szalay [3] (hereafter referred to as the SS model). The computer programs for using the SS model were also developed by the authors of the present work. Scatter points were obtained by Fews-Henshaw [10] method (open circles) and Nikezić-Yu [13] model (open squares). The computer program based on Fews and Henshaw model was also written by the authors of this work.

CONCLUSION

Considering that iterations were performed for the SS model in calculating the lengths of the axes, and numerical calculations are needed in other two models, very good agreement between these three independent models has been found for the considered ranges of incident energies and angles of the alpha particles. We thus conclude that the three models of the track development give compatible results.

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ТРОДИМЕНЗИОНАЛНИ МОДЕЛ РАСТА ТРАГА – ПОРЕЂЕЊЕ СА ДРУГИМ МОДЕЛИМА

У овом раду представљен је тродимензионални модел раста трага у чврстим детекторима трагова нуклеарних честица. Изведене су једначине зида трага у три димензије као и једначине контурних линија отвора трага за све типове трагова (тј. трагова са оштрим врхом и заобљених трагова). Израз за површину отвора трага такође је добијен у овом раду. Једначине се своде на добро познате изразе за малу и велику осу отвора трага у специјалном случају константне брзине нагризања.

Параметри трагова израчунати описаним моделом упоређени су са два модела раста трага приказана у литератури. Нађено је добро слагање између ова три независно постављена модела.