TRANSPORT THEORY AND SYSTEMS THEORY

by

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Received on March 1, 2005; accepted on March 29, 2005

The simulation of singular nonlinear transport equation is obtained via corresponding neutron or photon kinetic equation. The conditions for convergence of the nonstationary transport process toward the pure diffusion across the equilibriums are presented. For such purpose the method of transport scattering is exploited. The goal of these results is optimization of fusion fuels via neutron diagnostics.

Key words: transport equation, bifurcation, stabilization, neutron diagnostics, controlled fusion, tomography

INTRODUCTION

This paper is focused on the problems related to neutron diagnostic developments for the specific conditions of the plasma focus device operation. We shall see that we can obtain more general results for the neutron scattering than it is given by Lax-Phillips theory for acoustic waves. The various neutron diagnostic techniques used to determine the characteristics of the neutron fields of the plasma focus device have been developed with the following aims: identification of the relationship between neutron emission and plasma focus device operational parameters (*e. g.* electromagnetic quantities), and development of the plasma focus device as a fast neutron generator. We shall investigate the behaviour of neutron transport in different devices. Because nuclear power plants (NPP) are monitored by humans, their safe operation must be ensured by many operating procedures and safety margins. Various computer codes for diagnosing NPP abnormal operations have been developed. Our algorithm recognizes the prepared scenarios and it classifies them into groups. Global bifurcation problems mean that every solution is convergent to the set of equilibria if $t = 0$. As an example we can consider the following. The new feedback control of the neutron emission rate and the radiative power in the divertor has been performed [1]. The feedback control of neutron emission rate was demonstrated with controlling the heating power. Discharge operation will follow a sequence of the scenario phases. The model predicts the criterion for the loss of stability of the H-mode in fusion devices, and the observed Hopf bifurcation from the stable to the unstable H-mode gives the attracting limit cycle which behaves like the edge localized modes (ELM) plasma state [2]. The implementation of the feedback control concepts makes attainment of stationary plasmas possible. The future research will be concentrated on experimental verification of characteristics of tomographic measurements using a laboratory model of nuclear fuels.

Since the discovery of the H-mode, progress in the experimental and theoretical understanding of this improved confinement regime has been made. One aspect of the phenomenon is its bifurcation character. The plasma is monitored through the transverse light emission. Stability and reproducibility of the plasma have been derived out of corresponding measurements. Upon stabilization, the instability amplitude is reduced by factor 100, and the improved plasma confinement is observed (see [3]).

Scientific paper
UDC: 539.125.52
BIBLID: 1451-3994, *20* (2005), 1, pp. 50-58

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MATHEMATICAL INVESTIGATIONS OF BIFURCATIONS

The Cauchy problem for the nonlinear Boltzmann equation in the kinetic theory of neutron or photon has a unique classical solution f^{ε}, locally in time t at the interval $[0, \tau]$ independent of the mean free path $\varepsilon > 0$, if the initial distribution f_0

is sufficiently close to an absolute Maxwellian and is analytic in the space variable x. When $\varepsilon \quad 0, f^{\varepsilon}$ converges on $[0,\tau]$ to some f^0. For $t \quad [0,\tau]$ the limit f^0 is a local Maxwellian.

Theoretical work has included the development of a variety of techniques for determining the stability of local equilibria and bifurcations of local equilibria in general dynamical systems. Relative equilibria of equivariant dynamical systems are group orbits which are invariant under the flow of the dynamical system. In physical applications they typically correspond to the constant shape solutions which evolve by rotating or translating in space. The main goal is to explain the relevance of the spectral flow to the bifurcation theory. Let $\mathcal{F}: U \quad I \quad X$ be a continuously differentiable mapping defined on the product of a real interval I with a neighborhood U of the origin in a real Banach space X. We consider the bifurcations of zero equilibrium. Suppose that $\mathcal{F}(0,\lambda) = 0$ for all λ in I. Solutions of the equation $\mathcal{F}(f,\lambda) \quad 0$ of the form $(0,\lambda)$ are called trivial. A bifurcation point for solutions of the equation $\mathcal{F}(f,\lambda) \quad 0$ is a point λ^* in I, such that every neighborhood of $(0, \lambda^*)$ contains nontrivial solutions of this equation. Let $L_\lambda \quad D_f\mathcal{F}(0,\lambda)$ be the linearization of the mapping $\mathcal{F}_\lambda \quad \mathcal{F}(0,\lambda)$ at the trivial solution. By the implicit function theorem, bifurcation can occur only at the points where the operator L_λ is singular. The set of bifurcation points the mapping $\mathcal{F}$ is a closed subset of the set of singular points $\quad (L) \quad \lambda / L_\lambda$, L_λ is noninvertible of family L. However, in general, the set of bifurcation points of $\mathcal{F}$ may be empty even though the singular set $\quad (L)$ is very large.

Assume that the behavior of a device can be described by an differential equation $f \quad \mathcal{F}(f,\varepsilon t)$. The system is designed in such a way that f^* is an asymptotically stable equilibrium point, which corresponds to the desired behavior. Slow changes of the system's characteristics due to the ageing process can be modeled by a slowly time-dependent equation

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \mathcal{F}(f,\varepsilon t), \quad 0 \quad \varepsilon \ll 1 \tag{1}$$

where $\mathcal{F}(f, 0)$ describes the dynamics of the branch new device. As the eq. (1) is a non-autonomous differential equation which is difficult to solve, one is tempted to consider the one-parameter family of autonomous systems instead

$$\frac{\mathrm{d}f}{\mathrm{d}t} \quad \mathcal{F}(f,\lambda), \quad \lambda = \text{const} \tag{2}$$

One hopes that if the "quasistatic approximation" eq. (2) has a family of attractors depending smoothly on λ, then solutions of eq. (1) should be close at any given time t to the attractor of eq. (2) with $\lambda \quad \varepsilon t$ [4]. If $f^*(\lambda)$ is a family of asymptotically stable equilibria of eq. (2), then the solution of eq. (1) starting in a sufficiently small neighbourhood of $f^*(0)$ will, after a short transient, track the curve $f^*(\varepsilon t)$ at a distance of order ε. For the ageing device, this implies that we need not worry as long as the nominal equilibrium $f^*(\lambda)$ remains asymptotically stable. This naturally raises the question of what happens if the equilibrium $f^*(\lambda)$ undergoes a bifurcation at $\lambda = \lambda_0$, which may have a catastrophic consequence for the device. To avoid such a problem, one may try to control the system

$$f \quad \mathcal{F}(f,\lambda) \quad B \; u(f,\lambda) \tag{3}$$

stable when $\lambda = \lambda_0$. We have

$$f \quad \mathcal{F}(f,\varepsilon t) \quad B \; u_\varepsilon(f,\varepsilon t) \tag{4}$$

Since we wish to analyse eq. (4) on the time scale $\varepsilon \; ^1$, we introduce the slow time $\tau \quad \varepsilon t$ singular perturbation problem $\varepsilon\, \mathrm{d}f/\mathrm{d}\tau \quad \mathcal{F}(f,\tau) \quad Bu_\varepsilon(f,\tau)$. We have to modify the linearization A, A $\quad \partial\mathcal{F}/\partial f$ of $\mathcal{F}$ at the bifurcation point.

IRREDUCIBLE TRANSPORT SEMIGROUPS

Let $T(t)$, $t \quad 0$ be an irreducible C_0 – semigroup of positive operators on the ordered Banach lattice X, and let A be its infinitesimal generator. Let X and U be ordered by a normal positive cones X_+ and U_+ respectively (see [5]). We define a positive C_0 semigroup $T(t)$ on (X, X_+) to be irreducible if the only closed hereditary T-invariant subcones of X_+ are $\quad 0 \quad$ and X_+. The set of reachable states from the origin in arbitrary time t by means of nonnegative controls u is defined as

$$R_t \quad \int_0^t T(t \quad s)bu(s)\mathrm{d}s, \quad u \quad L^1[0,t\;],$$

$$u(s) \quad U \;, 0 \quad s \quad t \quad t \tag{5}$$

and the set of reachable states from the origin in arbitrary time by means of nonnegative controls u as

$$R_t \quad \bigcup_{t \quad 0} R_t \tag{6}$$

The system is approximately positive controllable if and only if $X_+ = VR_t$ (*i. e.* approximately controllable) where V denotes the closure. Since from X_+ is a normal positive cone follows the condition $X = V(X_+ - X_+)$, every approximately positive controllable system is approximately controllable. According to the result of [5] the following is equivalent for Banach lattice X: (1) $T(t)$ is irreducible, and (2) $T(t)$ is ergodic.

We have the following results. Let $T(\tau) = \exp(\tau \mathrm{A})$ be a positive C_0 – semigroup on an ordered Banach space (X, X_+): assume that (X, X_+) is a Banach lattice ordered by a normal positive cone X_+, then equivalent is:

(1) $\varepsilon\ \mathrm{d}f/\mathrm{d}\tau \quad \mathrm{A}f \quad b\,u_\varepsilon$ is approximately positive controllable for all $b \quad X \ / \ 0$, and

(2) $T(\tau)$ is ergodic.

Lemma 1. The following assertions are equivalent [6]:

(1) $T(\tau)$ is irreducible and ergodic,

(2) There exist ρ such that $(\rho I - \mathrm{A})^{-1}$ is irreducible and ergodic,

(3) $\varepsilon f \quad \mathrm{A}f \quad b\,u_\varepsilon$ is approximately positive controllable for all $b \quad X_+ \setminus \ 0$,

(4) $\varepsilon f \quad R(\rho, \mathrm{A})f \quad b\,u_\varepsilon$ approximately is positive controllable for all $b \quad X_+ \setminus \ 0$,

and some ρ, where X_+ is a closed convex cone in X, $\varepsilon > 0$.

The time dependent transport equation can be written as

$$\varepsilon \frac{\partial f}{\partial \tau} \quad \mathrm{A}f \tag{7}$$

where f represents the particle density per unit velocity space, and A is the Boltzmann operator. It is defined by the integrodifferential expression

$$\mathrm{A}f \quad v\,\mathrm{grad}\,f \quad \delta(x, v)f \quad \chi(x, v, v\ )f(x, v\ )\mathrm{d}v \tag{8}$$

The time-dependent transport equation has a unique solution $f(x, v, \tau)$, provided the initial distribution belongs to $D(\mathrm{A})$. The semigroup approach to the reactor problem leads to an abstract Cauchy problem in the ordered Banach space $X = L^1(D \quad V)$ where the configuration space D, $\mathrm{int}\,D \quad \phi$ is a subset of R^3. The velocity space V is a closed ball in R^3. The time dependent transport equation is given by

$$\varepsilon f \quad \mathrm{A}_0 f \quad \mathrm{M}_\delta f \quad \mathrm{K}_\chi f \tag{9}$$

where A_0 denotes the differential operator

$$\sum_{i\ 1}^{3} v_i \frac{\partial}{\partial x_i} \tag{10}$$

M_δ is the operator of multiplication by a positive measurable function $\delta : D \quad V \quad R$ and K is the integral operator given by

$$\mathrm{K}_\chi f \quad \chi(.,.,v\ )\,f(.,v\ )\mathrm{d}v \tag{11}$$

where $\chi : D \quad V \quad V \quad R$ is positive and measurable. δ and χ are called absorption coefficient and scattering kernel respectively.

There are different criteria when the semigroup $\exp(\tau \mathrm{A})$ is a positive irreducible semigroup [7, 8].

The pair (χ, δ) of the scattering kernel and the collision frequency is defined to be admissible if the following conditions hold:

(1) $\chi(x, v, v')$ is a nonnegative measurable function and $\delta(x, v)$ is a nonnegative measurable function in $L^1(D \quad V)$,

(2) $\delta(x, v)$, and

$\sigma_s(x, v) = \chi(x, v, v\ )\mathrm{d}v$ are bounded functions.

We consider also the conditions:

$$\text{(A) } \mathcal{F}(\delta) \quad \mathrm{ess}\,\chi, v\,\sup \int^{\infty} \delta(\chi \quad \tau v, v)\delta\tau \quad \infty$$
$$\text{(B) } \mathcal{F}(\sigma_s) \quad \infty \quad \infty \tag{12}$$

We now state the next lemma which establishes the similarity for the transport operator A to the collisionless transport operator A_0 (see [9]).

Lemma 2. Let (X, δ) be admissible. Suppose (A) and (B) hold.

Suppose also $\mathcal{F}(\sigma_s)\exp \mathcal{F}(\delta) < 1$. Then the wave operators exist, and possess the following properties:

$$(1)\ W_+(\mathrm{A}, \mathrm{A}_0)\, W_+(\mathrm{A}_0, \mathrm{A}) = W_+(\mathrm{A}_0, \mathrm{A})\, W_+(\mathrm{A}, \mathrm{A}_0) = I$$

$$(2)\ \mathrm{A} = W_+(\mathrm{A}, \mathrm{A}_0)\, \mathrm{A}_0\, W_+(\mathrm{A}, \mathrm{A}_0)^{-1}$$

We consider the two control systems Σ and Σ

$$\varepsilon f \quad \mathrm{A}_0 f \quad \lambda \mathrm{M}_\delta f \quad \mu \mathrm{K}_\chi f \quad b u_\varepsilon$$
$$\varepsilon f \quad \mathrm{A} f \quad b\, u_\varepsilon \tag{13}$$

for λ and μ are real positive parameters. For irreducible and ergodic positive semigroups we obtain the following result about reachability of the collisionless transport systems, actually diffusion systems, from the initial transport system.

Theorem 1. Let and Σ be two systems which are described by assumptions in lemma 2, and moreover let Σ be the irreducible and ergodic positive system. Then some approximately controllable collisionless transport system is reachable from the initial general transport system.

Proof. We construct auxiliary bounded systems Σ_1 and Σ associated via resolvents. Since the systems Σ and Σ are similar by lemma 2, it follows that the bounded systems Σ_1 and Σ are also similar. Actually, from $S\,\mathrm{A}\ S^{-1} = A$ we have $S\,\mathrm{A} = \mathrm{A}S$, $S\,\mathrm{A} - \rho S = \mathrm{A}S - \rho S$, $(\rho I - \mathrm{A})S = S(\rho I - \mathrm{A}\ )$, $S(\rho I - \mathrm{A}\ )^{-1} = (\rho I - \mathrm{A})^{-1} S$, and finally $S(\rho I - \mathrm{A}\ )^{-1} S^{-1} = (\rho I - \mathrm{A})^{-1}$, and conversely. These systems are positive controllable by lemma 1. Taking $\mathrm{A} = \mathrm{A}_0$ and putting λ, μ tend to zero, from formula (13) we obtain desired continuous path of approximately controllable systems because the operator $S = W_+(\mathrm{A}, \mathrm{A}_0)$ tends to identity operator. It is also true for

$$\lambda \mathrm{M}_\delta \quad \mu \mathrm{K}_\chi \quad 0 \tag{14}$$

The controllability of the systems will give the possibility of stabilization of such systems for irreducible quasicompact semigroups since such semigroups are ergodic and strongly stabilizable. After stabilization we have stationary regime and we can conclude as follows. Consider a system of linear equations

$$\mathrm{A}_2(\gamma)f \quad \mathrm{K}_\chi(\gamma)f \quad \lambda\, \mathrm{A}_2(\gamma)f \quad S(\gamma) \quad (15)$$

where the operator $\mathrm{A}_2(\gamma)$ describes the diffusion and absorption, $\mathrm{K}_\chi(\gamma)$ is the scattering operator, A_2 the fission, $S(\gamma)$ the external sources of neutrons, and f the neutron density in a given medium. The fundamental problem of reactor physics is to determine γ_0 so that eq. (15) with $S \quad 0$ and $\lambda \quad \lambda(\gamma_0) \quad 1$ has a nontrivial nonnegative solution f_0. This problem is a typical example for the applicability of the Frobenious comparasion theory. It can be shown that under certain assumptions the operator function $T(\gamma)$, where $T(\gamma) = [\lambda\, I \quad \mathrm{A}_2(\gamma)] \;^1\mathrm{K}_\chi(\gamma)$ is positive and has the property that for every $\gamma \quad 0$ and is monotonic in the sense that $T(\gamma) \quad T(\gamma\ )$ whenever $\gamma \quad \gamma$. It follows from Marek's comparison theory that the corresponding spectral radii $r[T(\gamma)]$ and $r[T(\gamma\ )]$ are similarly related $r[T(\gamma)] \quad r[T(\gamma\ )]$. If, moreover, $T(\gamma)$ is irreducible, then $r[T(\gamma)] \quad r[T(\gamma\ )]$ whenever $\gamma \quad \gamma \quad 0$, assuming $T(\gamma\ ) \quad T(\gamma)$ for $\gamma \quad \gamma$. Because of continuity of $T(\gamma)$ with respect to γ we conclude the following necessary and sufficient condition for the existence and uniqueness of the critical parameter γ_0, *i. e.* a value of parameter γ_0 for which $r[T(\gamma_0)] \quad 1;\quad r[T(0)] \quad 1$.

SCATTERING THEORY OF THE LINEAR BOLTZMANN OPERATOR

The existence of the Moeller operators is usually proved by use of the Cook theorem that is generalised to the case of a Banach space. In statistical mechanics transport phenomena of neutrons and photons are described by the linear Boltzmann equation. All results concerning the existence of the wave operators lead us naturally to the notion of A_2 – smoothness for $\mathrm{A}_2 = \mathrm{A}_0 - \mathrm{M}_\delta$.

Definition 1. Let A_2 be the generator of a C_0 group $e^{\tau \mathrm{A}_2}$ on a Banach space X. A linear operator K_χ is called A_2 – smooth with the constant $\alpha \quad 0$, if $\int^{\infty}_{\infty} \|\mathrm{K}_\chi e^{\tau \mathrm{A}_2} f\|_\chi \, d\tau \quad \alpha \|f\|_\chi$ holds for a dense set of vectors f in X (and hence for all f in X). If K_χ is A_2 – smooth with the constant $\alpha \quad 1$, then the wave operators $W\ (\mathrm{A},\mathrm{A}_2)$, $W\ (\mathrm{A}_2,\mathrm{A})$ exist. In [11] Voigt introduced the concept of a locally decaying system in the context of the transport theory to study the existence of wave operators. This concept means that for $V(\tau) \quad e^{\mathrm{A}\tau}$, $\lim_{\tau \quad \infty} \|V(\tau)f\|_{L^1(K \quad V)} \quad 0$ for bounded $K \quad R^M$. One can check that if $\mathrm{K}_\chi(0 \quad \mathrm{A}_2)\ ^1$ exists and if $r\sigma[\mathrm{K}_\chi(0 \quad \mathrm{A}_2)\ ^1] \quad 1$ then $V(\tau)$ is locally decaying, as $\tau \quad \infty$. One can also prove that $V(\tau)$ is locally decaying as $\tau \quad \infty$ if $\mathrm{K}_\chi(0 \quad \mathrm{A}_2)\ ^1$ exists and $r\sigma[\mathrm{K}_\chi(0 \quad A_2)\ ^1] \quad 1$. Under these conditions, the appropriate transport semigroup is irreducible. As we shall see later, this result holds under some conditions on absorber and the transport system will be stabilizable.

If both $\mathrm{K}_\chi(0 \quad \mathrm{A}_2)\ ^1$ exist and if $r\sigma[\mathrm{K}_\chi(0 \quad \mathrm{A}_2)\ ^1] \quad 1$, then s-$\lim_{\tau \ +\infty} V(\tau)S(\ \tau)$ exists, s-$\lim_{\tau\ +\infty} S(\ \tau)V(\tau)$ exists (for $V(\tau) \quad e^{\mathrm{A}\tau}$, $S(\tau) \quad e^{\mathrm{A}_2\tau}$), and conversely. Then $W\ (\mathrm{A}_2,\mathrm{A}_2 \quad \mathrm{K}_\chi)$ s-$\lim_{\tau\ +\infty} S(\ \tau)V(\tau)$ and $W\ (\mathrm{A}_2 \quad \mathrm{K}_\chi,\mathrm{A}_2)$ s-$\lim_{\tau\ +\infty} V(\ \tau)S(\tau)$ exist. Moreover, $\mathrm{A}_2 + \mathrm{K}\chi =$ $= W\ (\mathrm{A}_2 \quad \mathrm{K}_\chi,\mathrm{A}_2)\mathrm{A}_2 W\ (\mathrm{A}_2 \quad \mathrm{K}_\chi, \mathrm{A}_2)\ ^1$ and $W\ (\mathrm{A}_2 \quad \mathrm{K}_\chi,\mathrm{A}_2) W\ (\mathrm{A}_2,\mathrm{A}_2 \quad \mathrm{K}_\chi) \quad W\ (\mathrm{A}_2,\mathrm{A}_2 + \mathrm{K}_\chi) W\ (\mathrm{A}_2 \quad \mathrm{K}_\chi,\mathrm{A}_2) \quad I$. We can also conclude that $W\ (\mathrm{A},\mathrm{A}_0)\mathrm{A}_0 W_+(\mathrm{A},\mathrm{A}_0)\ ^1 \quad \mathrm{A}$ [12]. As we have pointed out, the power compactness of $\mathrm{K}_\chi(0 \quad \mathrm{A}_2)\ ^1$ may be replaced by $r\sigma[\mathrm{K}_\chi(0 \quad \mathrm{A}_2)\ ^1]$ is an eigenvalue of $\mathrm{K}_\chi(0 \quad \mathrm{A}_2)\ ^1$. A similar remark holds for $\mathrm{K}_\chi(0 \quad \mathrm{A}_2)\ ^1$.

In 1964 Adamjan and Arov showed the Lax-Phillips theory of acoustic scattering and Nagy-Foias theory about unitary dilatations to be equivalent [13]. The model of acoustic wave scattering is unitary equivalent to shift translation realization, but for the transport theory the appropriate models are mostly similar to translations, because we work in the Banach space.

Semigroups of translations on weighted function spaces on the real line or positive half-line are particularly well suited to serve as examples for understanding hypercyclicity since it is easy to determine the cases when these are hypercyclic [14]. A C_0 – semigroup of bounded linear operators $T(\tau)$ is called hypercyclic provided that there exists $f \quad X$ such that $\{T(\tau)f \mid \tau \quad 0\}$ is dense in X.

One of the most important features of the stability theory for linear systems is that it can be carried over to yield local results for nonlinear systems by linearization. It is known that a nonlinear semigroup whose Frechet derivative is an exponentially stable linear semigroup is locally asymptotically stable. Although hypercyclicity seems to be a property of an unstable semigroup, it can also occur in the critically unstable case, when trajectory grows slower than exponentially. From this point of view, it is not surprising that there are globally stable nonlinear systems, with hypercyclic (*i. e.* irreducible) linearizations.

GENERAL MODEL FOR TRANSPORT SCATTERING

Let us consider the nonlinear transport equation $\varepsilon f \quad \mathcal{F}(f, p)$ where $\mathcal{F}$ denotes a nonlinear operator that is differentiable on a dense domain of a separable Banach space $X \quad U$, where p is a control vector of parameters.

After linearization in some equilibrium neighbourhood the following equation holds true

$$\begin{aligned} \varepsilon \mathrm{d}f \quad & \mathrm{A}_1 \mathrm{d}f \quad B_1 \mathrm{d}p, \\ A_1 \quad & \partial \mathcal{F}/\partial f, \\ B_1 \quad & \partial \mathcal{F}/\partial p \end{aligned} \tag{I}$$

An equilibrium state of an appropriate semigroup evolution $T_1(\tau)$ with the infinitesimal generator A_1, is a state g such that $T_1(\tau)g \quad g$ for all $\tau \quad 0$. In the case of linear transport phenomena the equilibrium distribution will be a Maxwell-Boltzmann distribution. After linear transformation of coordinates in a zero equilibrium neighbourhood, we can write

$$\varepsilon f \quad \mathrm{A}_1 f \quad B_1 u \tag{II}$$

We shall derive the conditions under which the nonlinear transport equation will be exponentially stabilizable.

The linear distributed system of interest will be modeled by the following state-space form

$$\varepsilon \frac{\partial f}{\partial \tau} \quad \mathrm{A}f(\tau) \quad Bu_\varepsilon(\tau),$$

$$f(0) \quad f_0, \; f_0 \quad D(\mathrm{A}) \quad X \tag{16}$$

where A denotes the infinitesimal generator of a C_0 semigroup on the separable Banach state space X. Let U be the separable Banach space of control. We assume that the input operator B, $B{:}U \quad X$ is bounded and has a finite rank M, and $u(\tau)$ represent the inputs for M linear actuators. Thus

$$Bu_\varepsilon(t) \quad \sum_{i \; 1}^{M} b_i u_{i\varepsilon}(\tau) \tag{17}$$

where $u_{i\varepsilon}(\tau)$ are locally integrable control functions. Let $T(\tau)$ be a strongly continuous semigroup of bounded linear operators in the Banach space X with infinitesimal generator A. The following hold: there exists $\omega_0(\mathrm{A})$ such that

$$\omega_0(\mathrm{A}) \quad \lim_{\tau \quad \infty} \frac{1}{\tau} \log||T(\tau)|| \tag{18}$$

If $\omega \quad \omega_0(\mathrm{A})$ then there exist $M_\omega \quad 1$ such that $|T(\tau)| \quad M_\omega \mathrm{e}^{\omega\tau}$, $\tau \quad 0$. If $\omega_0(\mathrm{A}) \quad 0$ then eq. (18) yields the exponential asymptotic stability of the zero equilibrium of $T(\tau)$, $\tau \quad 0$.

The problem with an infinite dimensional system is that the spectrum of A does not always determine the exponential stability of $T(\tau)$, that is, equality does not always holds in $s(\mathrm{A}) \quad \omega_0(\mathrm{A})$, where $s(\mathrm{A})$ is the spectral bound and $\omega_0(\mathrm{A})$ is the spectral type

$$s(\mathrm{A}) \quad \sup[\mathrm{Re}\,\lambda | \lambda \quad \sigma(A)]$$

$$\omega_0(\mathrm{A}) \quad \inf\{\omega | \, ||T(t)|| \quad M_\omega \mathrm{e}^{\omega\tau}, \quad \tau \quad 0\} \tag{19}$$

The equality is so called a spectrum determined growth assumption. A consequence of the spectrum determined growth assumption is that if $\sup\,[\mathrm{Re}\,\sigma(\mathrm{A})] \quad \alpha$ then $||T_\tau|| \quad \mathrm{Me}^{\alpha\tau}$ for some $\mathrm{M} \quad 0$, $\alpha \quad 0$ and we say that $T(\tau) = T_\tau$(or A) is exponentially stable.

Let F be a feedback operator $\mathrm{F}: X \quad U$ and let $S_G(t)$ be the C_0 semigroup generated by $G = A + B\,\mathrm{F}$.

Definition 2. The autonomous linear system is exponentially stabilizable in case there is an operator F in $B(X, U)$ such that $||S_G(\tau)f_0|| \quad \mathrm{M}_{f0}\mathrm{e}^{\alpha\tau}, \alpha \quad 0$, $\tau \quad 0$ for all $f_0 \quad X$. (M_{f0} is constant depending on the initial data f_0.)

The semigroup approach to the reactor problem leads to an abstract Cauchy problem in the Banach lattice $X = L^1(D \quad V)$ where the configuration space D, $\mathrm{int}\,D \quad \phi$, is a compact convex subset of R^3. The velocity space V is a closed ball in R^3.

In this case we have $\varepsilon(\partial\mathcal{F}/\partial f) \quad \mathrm{A}_0 \quad \mathrm{M}_\delta \quad \mathrm{K}_\chi$ on a suitable dense subspace $D(\mathrm{A})$ of $L^1(D \quad V)$ of the transport operator $\mathrm{A}f \quad \mathrm{A}_0 f \quad \mathrm{M}_\delta f \quad \mathrm{K}_\chi f$.

We use the following result [15].

The collisionless transport operator $\mathrm{A}_0 = \sum_{i \; 1}^{3} v_i \, \partial/\partial x_i$, the streaming operator $\mathrm{A}_2 = \mathrm{A}_0 - \mathrm{M}_\delta$ and the transport or Boltzmann operator $\mathrm{A} \quad \mathrm{A}_0 \quad \mathrm{M}_\delta \quad \mathrm{K}_\chi$ each generates a strongly continuous semigroup $T_0(\tau)$, $S(\tau)$, $T(\tau)$ respectively. All semigroups consist of positive linear operators on $L^1(D \quad V)$; hence spectral bound and spectral type coincide.

The following holds:

$$\begin{aligned} s(\mathrm{A}_0) \quad & \omega_0(\mathrm{A}_0) \quad 0 \\ s(\mathrm{A}_2) \quad & \omega_0(\mathrm{A}_2) \quad 0 \\ s(\mathrm{A}) \quad & \omega_0(\mathrm{A}) \quad s(\mathrm{A}_2) \end{aligned} \tag{20}$$

The spectral bound $s(\mathrm{A}_2)$ of A_2 depends on the size of δ near $v = 0$; *e. g.* if δ is continuous at every point $(x, 0) \quad D \quad V$ then

$$s(\mathrm{A}_2) \quad \inf\{\delta(x,0){:}\, x \quad D\} \tag{21}$$

For every $\lambda \quad s(\mathrm{A}_2)$ the set $\{\lambda | \lambda \quad \sigma(\mathrm{A}), R_\mathrm{e}\,\lambda \quad \lambda\}$ contains only finitely many elements each of which is a pole of the resolvent $R(\cdot, A)$ with finite rank residue.

We have the following:

Theorem 2. Let A be a Boltzmann operator in $X = L^1(D \quad V)$. Let $\varepsilon(\partial f/\partial \tau) \quad \mathrm{A}f \quad Bu_\varepsilon$ be a linear distributed control system with an approximately controllable unstable part. If absorption rate δ is continuous at every point $(x,0) \quad D \quad V$ and $\inf\{\delta(x,0){:}\, x \quad D\} \quad 0$, then the control system is exponentially stabilizable [16].

Definition 3. The system (A_1, B_1) is said to be similar to (A, B) on $J = \; t_0, t$ if and only if there exists an invertible operator $S \quad L\,(X_1, X)$ such that $\mathrm{A}f = S\,\mathrm{A}_1 S^{-1} f$, $B = S\,B_1$ where:

$V[(D(SA_1S^{-1})\quad D(A\ )] = X$ is valid.

We have the following results:

Lemma 3. If (A_1, B_1) and $(A\ , B\ )$ are similar systems on J then (A_1, B_1) is controllable on J if and only if $(A\ , B\ )$ is controllable on J.

Definition 4. The system Σ_1 is said to be quasi-similar to Σ if there are two operators S and S such that:

(1) S^{-1}, S^{-1} with dense domain,

(2) $A_1Sf = S\,A\,f$ and $A\,S\,f = S\ A_1f$ with dense domain, and

(3) $B_1 = S\,B$ and $B = S\,B_1$ where:

$V[D(A\,S\ ^1)\quad D(S\ ^1A_1)]\quad X$, and

$V[D(A_1S\ ^1)\quad D(S\ ^1A\ )] = X$ is valid.

Suppose we are free to modify eq. (II) by setting $u = F_1f + v$ where v is a new external input. Thus for eq. (II) choosing a state feedback means choosing an operator F_1 and eq. (II) replacing it by

$$\varepsilon f \quad (A_1 \quad B_1F_1)f \quad B_1v \tag{22}$$

Suppose there is a pair $[F_1, S]$ such that the system $f = (A_1+B_1F_1)f + B_1v,\ y = G_1x$ has exactly the same input-output behaviour as the system Σ, $\varepsilon f = A\,f + B\,v,\ y = G\,x$. Then it is said that Σ can be (feedback) simulated by Σ_1.

The class of all approximately controllable systems which can be simulated by a given approximately controllable system Σ_1 is called the simulation orbit of Σ_1 and is denoted by $O\ \Sigma_1$.

In view of the controllability condition imposed on elements in $O\ \Sigma_1$, it is readily verified that $\Sigma\quad O\ \Sigma_1$ if and only if there is a pair F_1, S such that $S(A_1+B_1F_1) = A\,S$, $SB_1 = B$. It means that the systems Σ and $\varepsilon f = (A_1+ B_1F_1)f + B_1v$ are similar.

Theorem 3. Let us assume that we can choose the control parameters p in $\varepsilon f = \mathcal{F}(f, p)$ in such a way that the system Σ_1 is approximately controllable and $u = F_1f + v$ is such that the transport system $\varepsilon f = (A_1+ B_1F_1)f + B_1v$ is quasi-similar to the transport system Σ , so that A satisfies the hypotheses of theorem 2. Then the system Σ is in the simulation orbit of Σ_1. Also, the system Σ_1 is exponentially stabilizable [16]. The assumptions of theorem 3. hold for irreducible, quasicompact semigroups. We shall investigate the behaviour of stationary solutions of a branch of appropriate transport systems via strong stabilization and with the convergence toward diffusion.

The study of the scattering theory for transport phenomena was initiated by Lax and Phillips for the streaming group $U_0(\tau)f(x,v) = f(x-\tau v, v)$ in the context of the Hilbert space $X = L^2\ (R^3\quad S^2)$ [17]. As concrete examples of simulation orbit, we consider the absorbing transport equations. In [18] and [19] the representation theorem in the Banach space $X = L_1\ (R^n\quad V)$ for the transport semigroup $U(\tau)$ is generalized, which is governed by the transport equation

$$\frac{\partial f}{\partial \tau} \quad v \quad _x f \quad M_\delta f \quad K_\chi f \tag{23}$$

In some new functional space we have $V\quad \{v\quad R^n|0\quad v_{\min}\quad |v|\quad 1\}$ is the velocity space; the absorbing cross section δ and producing source function $\chi\ (x,\ v,\ v')$ are the positive functions in $L^\infty(R^n\quad V)$ and $L^\infty(R^n\quad V\quad V)$ having their supports as functions of x in a compact convex subset Ω of R^n. We can obtain the result for similarity of transport operators.

The classical scattering operator is W^*W [20]. By an admissible weight function on E we mean a measurable positive function ω satisfying $\omega(x\quad \tau v,v)\quad r(\tau)\omega(x,v)$ for all (x, v), where r is a positive function such that $\lim_{\tau\quad\infty} \inf r(\tau)\quad 0.\ L^1_\omega\quad \{f|\ ||f||_E\quad |f(x,v)|\,\omega(x,v)\,dxdv\quad \infty\}$. The Lax and Phillips representation theorem holds for the transport group $U(\tau)$ is $L^1_\omega(R^n\quad V)$.

Let $S(\tau)$ and $T(\tau)$ be two similar C_0 semigroups. If one of them is hypercyclic, then the other one is also hypercyclic. We consider the wave operators $W\ (A,A_0)\quad \text{s-lim}_{\tau\quad\infty} e^{\ \tau A}\ e^{\tau A_0}$, $W\ (A_0,\ A) =$ $\text{s-lim}_{\tau\ \ I\infty} e^{\ \tau A_0} e^{\tau A}$. Then $W\ (A,A_0)W\ (A_0,A)f = W\ (A_0,A)W\ (A,A_0)f$ for all $f\quad X$. It follows that the C_0 groups $e^{\tau A}$ and $e^{\tau A_0}$ are mutually similar under some assumptions on admissible pairs (δ, γ). For absorbing transport equation we obtain a concrete model of the simulation orbit for stable systems. Control problem in fusion devices has initiated the search on adaptive controllers that adapt their parameters to changing conditions. Therefore, the topology of the solution set may change during the computation. The wave operators exist and have the explicit forms. Let

$$\inf_{(x\,v)\ \ R^n\ \ V} \delta(x,v)\quad \delta\quad 0 \tag{24}$$

holds. The following theorem is valid (see [20]).

Theorem 4. Assuming eq. (24), then there exists a weight function ω such that in $L^1_\omega(R^n\quad V)$ the C_0 group $S(\tau)$ is hypercyclic and satisfies $||S(\tau)||_\omega\quad 1, \tau\quad 0, ||S(\tau)f||_\omega\quad e^{(\alpha\ \ \delta)\tau}||f||_\omega\quad ||f||_\omega$. This group is not uniformly bounded, for $\tau \le 0$; hence we can not use the Cook's Lemma for the existence of $W\ (A,\ A_2)$ and $W\ (A_2,\ A)$ [11].

SIMULATIONS OF BIFURCATIONS OF TRANSPORT PHENOMENA

Singularly perturbed systems of an abstract ordinary differential equation can often be written in the form

$$\varepsilon f \quad \mathcal{F}(x, v, \tau, \varepsilon) \tag{25}$$

where x and v are vectors x (0) and v (0) are prescribed, 0 ε 1 and usually ranges over some finite subinterval of τ 0. The numerical integration of eq. (25) involves difficulties because the solution often features a narrow initial boundary layer region of rapid change. By using a combination of numerical and asymptotic techniques, we can obtain an effective solution method. Treatments may, of course, break down whenever eq. (25) becomes singular and $\mathcal{F}(x, v, \tau, 0)$ fails to have a unique solution. In the classical theory, it is supposed that the Jacobian of $\mathcal{F}(x, v, \tau, 0)$ has stable (*i. e.* negative real part) eigenvalues and a reduced problem has a unique solution of the interval of interest.

Singular behaviour at $\varepsilon = \tau = 0$ describes the initial layer of the solution of the Boltzmann equation, and the limit f plays the role of the other solution in the theory of singular perturbations. The Cauchy problem for the Boltzmann equation is written as

$$\frac{\partial f}{\partial \tau} \quad \frac{v}{\varepsilon} \quad f \quad \frac{1}{\varepsilon}[\ \mathrm{M}_{\delta} \quad \mathrm{K}_{\chi}\ ]f,\ f/_{\tau\ 0} \quad f_0 \qquad (26)$$

When ε 0 eq. (26) raises a singular perturbation problem. To find the corresponding reduced problem, suppose f^{ε} have a limit f^{0} and $\varepsilon(f_{\tau}^{\varepsilon}$ v/ε ${}_{x}f^{\varepsilon})$ 0 as ε 0. Then letting ε tend to 0 in eq. (26), we find

$$(\ \mathrm{M}_{\delta} \quad \mathrm{K}_{\chi})f \quad 0 \qquad (27)$$

Unique solutions to eq. (27) are Maxwellians. As ε 0, f_0 should be chosen indefinitely close to the Maxwellians. Local solutions exist on the interval $[0, \tau^{\varepsilon}]$, $\tau^{\varepsilon} > 0$, depends on ε as well as f_0 may tend to 0 with ε. We shall derive convergence toward diffusion case for the appropriate transport control systems with feedback.

We have a notion of singular hyperbolic set if at every point x there exists decomposition $T_x M = E_x$ F_x of the tangent space into two subspaces E_x and F_x such that the linear flow contracts exponentially, and is exponentially volume expanding restricted to F_x: :det $(D_{\varphi}{}^{\tau}(x)|F_x)$ $C\mathrm{e}$ ${}^{\lambda\tau}$ for all $\tau > 0$, for the 3-dimensional flow. It should be dominated by the one in the E_x direction: $||D\varphi^{\tau}(x)\mathrm{e}||/||D\varphi^{\tau}(x)f||$ $C\mathrm{e}$ ${}^{\lambda\tau}$ [21].

If there are equilibrium points in Λ, they should all be hyperbolic (no eigenvalue with zero real part). If U' is open in X and $T(\tau)$ U' is contained in U' and relatively compact for all sufficiently large τ, then Λ ${}_{\tau\ 0}T(\tau)U$ is a compact attracting set, with fundamental neighborhood U'. Let $T(\tau)$ be a dynamical system, *i. e.* a group or semigroup of maps X X parametrized by a discrete or continuous time τ. There often exist subsets Λ of X which attract neighboring points f, meaning that $T(\tau)f$ tends to Λ when τ ∞. Such subsets Λ are called attracting sets or attractors. In the simplest case Λ is an attracting fixed point or periodic orbit. In the case of transport equation the fixed points will be Maxwellians. If an attracting set consists of a number of disjoint invariant pieces, one would like to consider each piece as an attractor, removing irrelevant points like (perhaps) wandering points. Examples of irreducibility conditions are: positive transitivity (there is x Λ such that the set of limit points of $T(\tau)f$ is Λ) or existence of an ergodic measure with support Λ.

One of the most fundamental concepts in the study of dynamical systems is that of the rate of growth of a quantity with time. Perhaps the most familiar example of this is the rate of expansion or contraction of infinitesimal perturbations (*i. e.* of tangent vectors). Recall that if T : X X is a diffeomorphism of a compact manifold X, x M and f T_xM, then this is given by $\lambda(x, f)$ $\lim_{n\ \infty}(1/n)$ $\log||D_x T^n f||$ whenever the limit exists. If μ invariant measure is ergodic, then λ takes on only a finite number of possible values, called Liapunov exponents. In many problems it is important to distinguish between uniform and non-uniform convergence of growth rates. An invariant set is hyperbolic if the tangent space admits a continuous decomposition into expanding and contracting subspaces $||D_x T^n f||$ $c\mathrm{e}^{\lambda n}||f||$; $||D_x T^n f||$ $(1/c)\mathrm{e}$ ${}^{\lambda n}||f||$.

For generally uniformly hyperbolic diffeomorphisms (Sinai, Ruelle, Bowen) have shown that there are finitely many probability measures μ_1, μ_N such that lim $(1/n)$ ${}_{i}^{\infty}{}_{0}\,\varphi[T^{i}(f)]$ $\varphi\mathrm{d}\mu_i$ for all continuous $\varphi : M$ R and for points f in a positive Lebesque measure subset $B(\mu_t)$ of M, $i = 1, \ldots \ell$. These measures are called SRB measures and the sets $B(\mu_t)$ are the basins of these measures [22].

We consider some kind of evolution of infinite dimensional nonlinear systems near equilibriums. As a model, we take quasicompact, irreducible and absorbing transport semigroups.

In acoustic and electromagnetic problems, the method of approximating the field scattered by a moving body is to calculate the stationary field scattered by the body at each time t. This yields a time-dependent sequence of stationary fields referred to as the quasi-stationary fields [23]. In the problem of controlled fusion we have the situation of the controlled path of stabilizable transport systems. We consider transport equation f $\mathrm{A}_0 f$ $\mathrm{M}_{\delta}f$ $\mathrm{K}_{\chi}f$ $B\,u(t)$. We shall assume that f_1 is the solution of the stationary transport equation $\mathrm{A}_0 f_1$ $\mathrm{M}_{\delta}f_1$ $\mathrm{K}_{\chi}f_1$ $B\,u(t)$ 0. The error is $E(t)$ $f(t)$ $f_1(t)$.

We shall obtain the possibility of stationary solutions if we also take control mechanism in the bifurcation phenomena. Let us consider a singularly perturbed problem of photon or neutron transport in a time-dependent region [24]

$$\frac{df}{d\tau} \quad \frac{A_0}{\varepsilon} \quad \frac{1}{\varepsilon} M_\delta \quad \frac{1}{\varepsilon} K_\chi \; f(\tau) \quad B u_\varepsilon(\tau) \qquad (28)$$

Since ε is a very small parameter, it is reasonable that the time derivative in eq. (28) is small compared with other terms. The goal is therefore to compare the solution of such an equation with the solution $f_1(\tau)$ of a simpler one, called the quasi-static approximation, which satisfies the following equation

$$\frac{A_0}{\varepsilon} \quad \frac{1}{\varepsilon} M_\delta \quad \frac{1}{\varepsilon} K_\chi \; f_1(\tau) \quad B u_\varepsilon(\tau) \quad 0 \qquad (29)$$

obtained by deleting the time derivative term in the original problem $(1/\varepsilon)(A_0 \quad M_\delta \quad K_\chi) f(\tau)$ $B u_\varepsilon(\tau) \quad 0$ [24].

It could be done in a better way by including control and stabilization when it is possible. If the transport semigroup is quasicompact, irreducible (hypercyclic), then the appropriate system is stabilizable. It means that with the appropriate control in each ergodic subclass we can reach stable stationary regime.We have $df/d\tau \quad [(A_0/\varepsilon) \quad (M_\delta/\varepsilon) \quad (K_\chi/\varepsilon)]$ $f(\tau) \quad B u_\varepsilon(\tau)$. For $u_\varepsilon(\tau) \quad F_\varepsilon f$ it follows $f \quad (1/\varepsilon)(A_0 \quad M_\delta \quad K_\chi) f \quad B F_\varepsilon f$ where f tends to some equilibrium. The absolute error E between the exact control system and stationary approximation tends to zero after taking the stabilization. From this it follows possible description of optimal work of transport devices. The errors will tend to zero for the systems stabilizable in a finite time intervals $[\tau_0, \tau_1]$, $[\tau_1, \tau_2]$, ...$[\tau_{N-1}, \tau_N]$.

We use the method of quasistatic approximations in combination with stabilization.

As a consequence we have the next theorem.

Theorem 5. Let $\varepsilon f \quad \mathcal{F}(f, p)$ be a nonlinear transport system that is in the simulation orbit of irreducible, absorbing quasicompact neutron kinetic equation. Then there exists a sequence of times τ_1, τ_2, ...τ_n with convergence to diffusion, of solutions of stationary transport problem.

When evolution of transport system changes slowly in relations with photon or neutron transport, the tomography, based on scattering operators, is possible.

As a conclusion, we have that here obtained model could be useful for numerical simulations of the nonlinear phenomena with the better degree of accuracy then one which can be achieved by experiments. The same result holds if transport phenomena can be found as simulation orbit of transport equation of the type (7) under condition $\alpha \quad \delta \quad 0$.

CONCLUSION

In the aim to obtain the possibility of uniform stabilization of a control system we used the method of adaptive controllers for nonlinear transport systems. Instead of theoretical results of the type of Trotter-Kato theorem for linear semigroups we used the method of neutron tomography for obtaining the desired stabilization with considering the wave operators in vertical and horizontal direction on locally linearizable pieces. This method can be extended on periodical cases (or, more generally adaptively recurrent with avoiding the method of Cook). On this way, it opens the possibilities for experimentation with the influence of real actuators on different devices in nuclear technology and radiation science.

For more precise mathematical treatment of some aspects of ions transport behaviour see the refs. [25, 26].

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Данило РАСТОВИЋ

ТРАНСПОРТНА ТЕОРИЈА И ТЕОРИЈА СИСТЕМА

Коришћењем одговарајуће кинетичке једначине неутрона, или фотона, симулирана је сингуларна нелинеарна транспортна једначина. Посредством равнотежних стања приказан је услов конвергенције нестационарног транспортног процеса према чистој дифузији. У ту сврху коришћена је метода транспортног расејања. Циљ овог истраживања је оптимизација фузионог горива помоћу неутронске дијагностике.

Кључне речи: транспортна једначина, бифуркација, стабилизација, неутронска дијагностика, контролисана фузија, томографија