DIRECT DISCRETE METHOD AND ITS APPLICATION TO NEUTRON TRANSPORT PROBLEMS

by

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The objective of this paper is to introduce a new direct method for neutronic calculations. This method, called direct discrete method, is simpler than the application of the neutron transport equation and more compatible with the physical meanings of the problem. The method, based on the physics of the problem, initially runs through meshing of the desired geometry. Next, the balance equation for each mesh interval is written. Considering the connection between the mesh intervals, the final discrete equation series are directly obtained without the need to pass through the set up of the neutron transport differential equation first. In this paper, one and multigroup neutron transport discrete equation has been produced for a cylindrical shape fuel element with and without the associated clad and the coolant regions, each with two different external boundary conditions. The validity of the results from this new method is tested against the results obtained by the MCNP-4B and the ANISN codes.

Key words: neutrons, transport equation, direct diserete method, ANISN code, MCNP-4B code

INTRODUCTION

A control volume is usually chosen for solving the physical problems on hand, and the production, absorption, input and output terms are written for it. Then, if the control volume approaches zero, the relevant differential equation can be derived. This equation – with its initial and boundary conditions – express the mentioned physical phenomena in mathematical formulation. The derived differential equation is not usually easy to solve except for simple and symmetrical geometries. Therefore, numerical methods are to be used. In this regard, the continuous parameters must be converted to discrete parameters to produce algebraic equation series [1].

The main goal in neutronic field is calculation of neutron population distribution (neutron flux distribution) in a reactor core. The balance equation for the neutron population distribution inside a reactor core is:

$$\frac{\partial n}{\partial t} \quad P \quad L \tag{1}$$

where: n is the neutron population in time t; $\partial n/\partial t$ – change in neutron population versus time, P – neutron production, and L – neutron loss.

Neutron population in this equation depends on the following seven factors (x, y, z, E, Ω_x, Ω_y, t). Neutron production consists of fission neutrons, extraneous sources and scattering from other energy and angle intervals to a defined energy and angle intervals. Neutron loss also includes absorption, leakage from reactor core and scattering to other energy and angle intervals. Further mathematical calculations lead to the following integro-differential neutron transport equation [2]:

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$$\frac{\partial}{\partial t}\int_V n(r,E,\Omega,t)\,dVdEd\Omega$$

$$\int_V S(r,E,\Omega,t)\,dVdEd\Omega \qquad \int_S dS\nu\Omega n(r,E,\Omega,t)\,dEd\Omega$$

$$\int_V dV \int_{4\pi} d\Omega \int_0^{\infty} dE\,\nu \quad {}_s(E \quad E,\Omega \quad \Omega) n(r,E,\Omega,t) dEd\Omega -$$

$$\int_V \nu \quad {}_t(r,E) n(r,E,\Omega,t) dVdEd\Omega \qquad (2)$$

It is apparent that the above equation is not easy to solve even for simple and symmetrical geometries. This equation is usually reduced to a simple one using different numerical methods. Few of the approximate methods devised to make this equation more amenable to acceptable solutions are as follows [3]: (a) collision probability (Pij), (b) discrete S_N method, and (c) P_L approximation and diffusion equation.

Other probabilistic methods such as Monte Carlo and nodal methods have also been presented for neutron flux distribution calculations. While each of these methods has its own merits, they are not free of shortcomings. When applying each of the above methods to solve the neutron transport equation, mathematical tools are given so much concern that the physics of the problem is usually lost in between. Also, sometimes, new mathematical parameters (such as diffusion coefficient and diffusion length in production of diffusion equation) that have no specific physical meanings are defined and are solely artificial mathematical parameters. In the next section, some of the shortcomings of differential formulation are listed.

RESTRICTIONS IN APPLYING DIFFERENTIAL FORMULATION

Some intricacies in applying differential formulation have already been stated. Here, some more restrictions in applying this formulation are listed.

(1) Physical variables can be classified into two main categories: global quantities and field functions.

Global quantities are directly measurable in the laboratory; therefore, they must be physical and realizable parameters such as *mass*, *internal energy* and *neutron population*.

The corresponding field functions are derived from these global variables by a limiting process and are called *mass density*, *energy density* and *neutron population density*.

Differential formulation of physical laws requires the conversion of global variables into field functions by the limiting process applied to the line, surface and volume to get densities and to the time interval to get rates.

(2) The analytical solutions of differential equations are normally possible for smooth boundaries. This condition is not commonly met in practice. Therefore, numerical treatment is usually used.

(3) Usually sources are concentrated in small regions like the heat spot of a laser beam or a point neutron source. The differential formulation leads to considering pointwise concentrated sources, which are unphysical, instead of sources with given intensity concentrated in a small but finite area. In order to overcome this problem, the Dirac generalized function is introduced.

(4) In addition to few of the mentioned shortcomings attributable to differential formulation, there is one other major drawback and that is its seldom adaptability to analytical solutions. As a result, one should resort to numerical methods, such as the finite difference method, the finite element method, the weighted residual method, the least square method [4], *etc*., in order to be able to discretize the differential equations and thus produce a finite set of algebraic equations.

With these introductory remarks we are now in a position to pose the following questions:

– Why use the differential formulation against all these restrictions and complications?

– Is the differential formulation the only way to formulate a physical phenomenon?

– Is it possible to directly obtain a discrete form of physical laws without a compulsory passage into the differential formulation?

The answers to all these questions – with the notable advance in speed of calculations and the volume of the memory of today's computers – may be given by introducing the new *direct discrete method (DDM)*. This method is much simpler and more compatible with the meanings of the physical laws when compared with the customary and widespread differential formulation method. This method has been successfully applied to newtonian mechanic [5], electrostatic, electrodynamics [6], heat transfer [1], and fluid mechanics. The method is further developed to adapt it to the general neutronic calculations.

GENERAL REMARKS ON DDM FORMULATION

The question here is how to produce discrete formulations for a physical problem. Three major

steps are envisioned to transform the physical problem into the DDM model:

(1) Identification of the global variables of the specific problem on hand: as mentioned before, physical variables are classified into two categories. Differential formulation uses field functions, which are spurious and unphysical parameters. DDM uses global variables, which are real and physical parameters. The global variables should be identified for the defined physical field first. *Neutron population* is a global variable in a neutronic field.

(2) Adoption of a suitable meshing scheme for the specified geometry: coordinate systems are the essential tools to derive and solve the differential equations. In differential formulation, a coordinate system is usually chosen and then the integrals and derivatives are discretized with notice to the chosen coordinate system. As a result, the differential equations change to a set of algebraic equations. Similarly, in the DDM method, a suitable meshing scheme should be adopted such as triangular, rectangular, cylindrical or spherical mesh depending on the given geometry and its dimensions. For instance, in pin-cell calculations, it is better to use cylindrical meshing scheme considering the fact that fuel elements are usually cylindrical in shape.

(3) Formulation of the balance equation for each mesh interval: the balance equation should be written for each of the generated mesh intervals considering the physics of the problem. It should be mentioned that due to dependence of each mesh interval equation on its neighboring mesh interval equations, the set of the generated DDM equations must therefore be solved simultaneously.

Finally, it is important to note that the DDM formulation can transform into the differential formulation using the limiting process. Figure 1 shows the main differences between these two distinct approaches or methods.

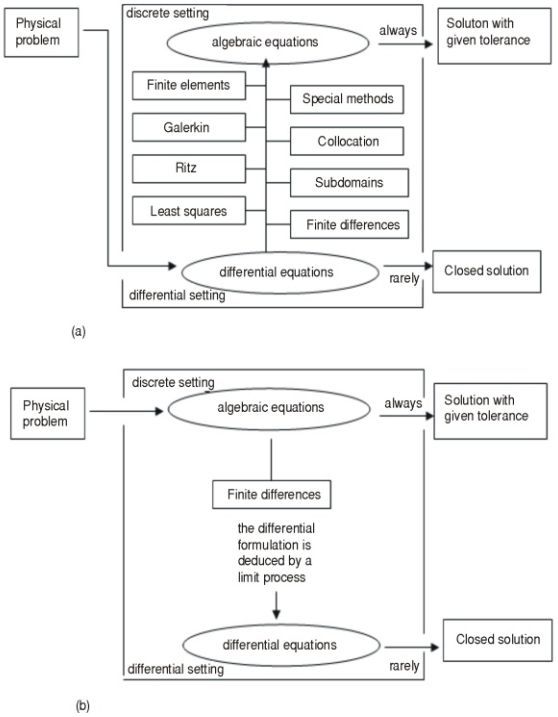

Figure 1. (a) Traditional approximate methods are based on discretization of differential equations; (b) the direct discrete method (DDM) permits to directly obtain the discrete form of physical equations from physical facts

APPLICATION OF DDM TO NEUTRONIC FIELDS

As previously mentioned, to apply DDM to a physical problem, first the physics and the geometry of the problem have to be completely known and its global variables identified. Therefore, neutron population N is defined as a global variable in a neutronic field. As an example, let us consider a cylindrical fuel element with volume V and surface S. Next, assume a time interval t selected under some special conditions. The neutron balance equation can now be written for the existing neutrons in this position-time element based on the events which might happen to these neutrons inside the fuel element.

The following essential assumptions have been made in deriving the neutron discrete equations:

(1) One-group energy;

(2) Uniform distribution of the materials occupying the regions of the various mesh intervals of the volume element – the dimensions of the mesh intervals are normally so small that make this assumption acceptable;

(3) Uniform distribution of neutron population in each mesh interval;

(4) The rates of the entering and the exiting neutrons across the various surfaces of the mesh intervals are assumed to be constant;

(5) Limit the time interval t so as to allow only one neutron interaction;

(6) Neutron-neutron collision is neglected;

(7) The static-state case is considered.

Finally, let us write down the general balance equation independent of the shape, dimension, and the material make up of the element under study:

$$P(V,t) \quad A(V,t) \quad I(S,t) \quad O(S,t) \quad 0 \quad (3)$$

We have in the above equation: P – neutron production, A – neutron absorption, I – neutron input, O – neutron output, V – the volume of the element, S –

the peripheral surface of the element, and t – the observation time interval.

Each of the terms identified above will later be explicitly construed, using the neutronic global variable (N).

PRODUCTION OF NEUTRON BALANCE EQUATION (USING PROBABILITIES)

In this section, we will adapt the above balance equation to neutronic calculation. To start out, we shall divide the neutron population into two separate entities. The *primary (already available)* neutrons within the different mesh regions and the *secondary (entrant)* neutrons that enter into different meshes through their corresponding surfaces. It will be seen later, that there are in fact no substantial differences between these two groups of neutrons and that this division simply comes to be handy when deriving the discrete equations.

Primary neutrons

A cylindrical shape fuel element is assumed with a population of neutrons already inside it. Next we will investigate the fate of these neutrons during the observation time interval t. Also we assume that no neutrons enter the volume through the boundaries at this stage. These neutrons may participate in the following reactions:

(1) Neutrons may participate in absorption reaction:
- absorbed neutrons may cause fission,
- neutrons produced from fission may escape from the volume element,
- neutrons produced from fission may remain in the volume element, and
- absorbed neutrons may just be captured;

(2) Neutrons may participate in scattering reaction:
- neutrons after scattering may escape from the volume element, and
- neutrons after scattering may remain inside the volume element;

(3) Neutrons may remain in the volume element without any reaction;

(4) Neutrons may escape from the volume element without any reaction.

"Escape after scattering" and "escape without any reaction" are not differentiated, because in either case a neutron is lost from the mentioned volume element. On the other hand, neutrons set out to move in a certain direction with a finite speed and experiencing no interactions do not necessarily all get the chance to leave the volume element in the finite observation time interval t. Only neutrons, which are close to the boundary of the element, can escape from the volume element. If the speed of neutrons is assigned to be v and the time interval as already introduced is assumed to be t, then the furthest distance that a wandering neutron can travel is:

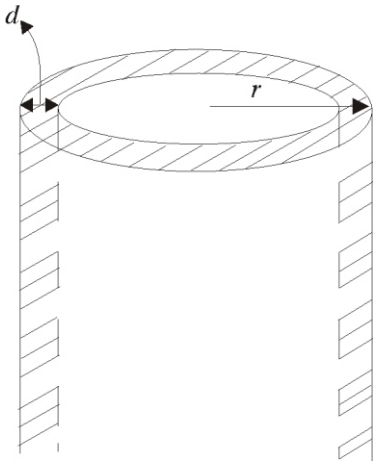

Figure 2. Cylindrical element and its boundary layer

$$d \quad v\ t \tag{4}$$

The region realized by this distance which is adjacent to the surface of every mesh interval, is named the boundary layer thickness, fig. 2. It is obvious that neutrons lying within this boundary layer have the chance of escaping the region. Considering the definition of this boundary layer, the primary neutrons may be categorized into two different groups:

(1) Neutrons in the internal zone with an escape probability equal zero;

(2) Neutrons in the boundary layer that have the chance of escaping the region.

Since the neutron population distribution in each mesh interval has already been assumed as uniform, hence the ratio of neutrons in each of the above mentioned two regions to the total number of neutrons equals the ratio of the volume of the respective regions to the total volume of the volume element. Two new parameters, a (neutrons' *internal zone fraction*) and b (neutrons' *boundary layer fraction*) are defined as:

$$b \quad \frac{N_b}{N} \quad \frac{V_b}{V} \quad \frac{[\pi r^2 \quad \pi(r \quad d)^2]h}{\pi r^2 h}$$

$$\frac{d^2 \quad 2dr}{r^2} \quad \frac{2d}{r} \tag{5}$$

$$a \quad \frac{N_i}{N} \quad \frac{V_i}{V} \quad \frac{\pi(r \quad d)^2 h}{\pi r^2 h} \quad \frac{(r \quad d)^2}{r^2} \tag{6}$$

where: N_b is the number of neutrons in the boundary layer, V_b – volume of the boundary layer, N_i – number of neutrons in internal zone, V_i – volume of internal zone, N – number of the total neutrons in the volume element, and V – total volume of the volume element.

However, d^2 is neglected against $2rd$ in the above equation. Consequently, the number of neutrons in the internal zone will be equal to aN and the number of neutrons in the boundary layer will be bN.

Neutrons in internal zone *(aN)*

The escape probability for this group of neutrons is zero and the reactions that they may undergo are the following:

– probability of neutrons not absorbed in the time interval t:

$$\exp(\ _a vt)$$

– probability of neutron absorption in the time interval t:

$$1\ \exp(\ _a vt)$$

– probability of fission (if they are absorbed):

$$\frac{\ _f}{\ _a}$$

– probability of capture (if they are absorbed):

$$\frac{\ _c}{\ _a}$$

Neutrons in boundary layer *(bN)*

The escape probability for this group of neutrons is not zero. These neutrons, on the average, have less time available to them, compared to the other group of neutrons, to participate in different reactions. It is probable for these neutrons to escape from the boundary layer while in their random movement. The average time period available to the boundary layer neutrons is considered as t , which will be discussed in more detail in the next section. For this group of neutrons, the following results are obtained:

(1) Probability of neutrons not absorbed in the observation time interval t :

$$\exp(\ _a vt^*)$$

– escape probability from the boundary layer (if they are not absorbed): P;
– residence probability in the boundary layer (if they are not absorbed): $1 - P$;

(2) Absorption probability in the observation time interval t :

$$1\ \exp(\ _a vt^*)$$

– fission probability (if they are absorbed):

$$\frac{\ _f}{\ _a}$$

– escape probability for neutrons produced from fission: P;
– residence probability in the element for neutrons produced from fission: $1–P$;
– capture probability (if they are absorbed):

$$\frac{\ _c}{\ _a}$$

By lumping all of the above and the previous terms together, the following explicit results for the production, absorption and the output sentences of the primary neutrons are obtained:

$$p(V,t)\quad N\{a[1\ \exp(\ _a vt)]$$

$$b[1\ \exp(\ _a vt^*)]\}\frac{\upsilon\ _f}{\ _a} \qquad (7)$$

$$A(V,t)\quad N\{a[1\ \exp(\ _a vt)]$$

$$b[1\ \exp(\ _a vt^*)]\} \qquad (8)$$

$$O(S,t)\quad NPb\ [1\ \exp(\ _a vt^*)]\frac{\upsilon\ _f}{\ _a}$$

$$\exp(\ _a vt^*) \qquad (9)$$

Calculation of the neutron escape probability *(P)*

Here, isotropic scattering and a uniform distribution of neutron populations are assumed. Admittedly, by accepting these assumptions, the accuracy of the model is somewhat reduced. With these remarks in

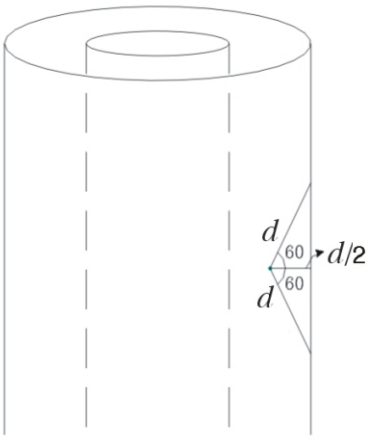

Figure 3. Escape probability calculation for boundary layer neutrons

mind, one can imagine that the population of neutrons in the mesh volume is located at half distance from the surface of the mesh interval. If it is taken that the maximum distance that the neutrons can travel until they escape from the region is d (d being the radius of a sphere centered at the point where the escape calculation is to be made, refer to fig. 3 for further clarification), then the escape probability is calculated to be:

$$P \quad \frac{2\pi \int_0^{60} \sin\theta \, d\theta}{4\pi} \quad \frac{1}{4} \tag{10}$$

Calculation of the time interval (t) for the boundary layer neutrons

Since the average time period available to the boundary layer neutrons which do not escape the volume element is t, and the average time period available to the boundary layer neutrons which escape the volume element is $t/2$, then one can easily calculate the average time available to the entire population in that layer using the above obtained result for the escape probability as follows:

$$t^* \quad \frac{1}{4}\frac{t}{2} \quad \frac{3}{4}t \quad \frac{7t}{8} \tag{11}$$

Considering the above result, we can without any severe approximation and for simplicity assume t as equal to t.

The secondary (entrant) neutrons

While this group of neutrons and the primary neutrons hold a lot of similarities, they, however, differ in two distinct aspects:

(1) The primary neutrons have a spatial angle distribution between 0 and 4 steradian, whereas the entering neutrons have a direction towards the volume element; therefore a spatial angle distribution between 0 and 2 steradian;

(2) The average time period available to these neutrons to participate in any reaction within the boundary layer equals half of the average time period available to primary neutrons.

Let us identify the entering neutrons into the volume element by the parameter I. Using this parameter one can now write down the relevant production, absorption and the output terms arising from this group of neutrons as follows:

$$P_i(V,t) \quad I \; 1 \quad \exp \quad {}_a \nu \frac{t}{2} \quad \frac{\upsilon \quad f}{{}_a} \tag{12}$$

$$A_i(V,t) \quad I \; 1 \quad \exp \quad {}_a \nu \frac{t}{2} \tag{13}$$

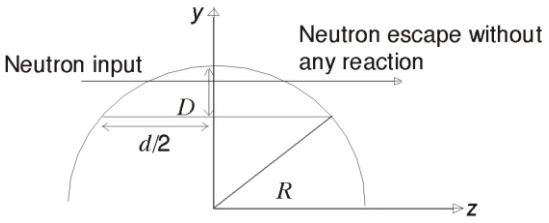

Figure 4. Escape probability of neutrons without any reactions

where the index i corresponds to the above defined parameter I.

Now, for the calculation of the output term O_i, two groups of neutrons should be considered:

(1) The input neutrons which do not participate in any reaction after entering the volume element. This group of neutrons can travel the maximum distance d and then escape the volume element (fig. 4). Therefore, a fraction of neutrons that enter from the top of the line d and do not participate in any reaction can escape from the element. C_1 is defined as the fraction of neutrons that have this condition (entering from the top of the line d) as follows:

$$C_1 \quad \frac{D}{R} \tag{14}$$

Since I is the input neutrons into the mesh region, then IC_1 is the total number of neutrons entering the boundary layer. In this regard, N_1 (neutrons which enter the boundary layer in one specific dimension) is defined as:

$$N_1 \quad \frac{1}{4\pi} IC_1 \tag{15}$$

The number of neutrons entering the boundary layer, defined by the differential element dy, and escaping the region without any reaction is:

$$N_1 \frac{dy}{D} \exp(\; 2 \quad {}_t Z) \tag{16}$$

Furthermore, the total output sentence for this group of neutrons is:

$$O_{i1} \quad N_1 \frac{dy}{D} \exp(\; 2 \quad {}_t Z) \tag{17}$$

From fig. 4, we have:

$$y^2 \quad Z^2 \quad R^2 \quad 2y\,dy \quad 2Z\,dZ \tag{18}$$

hence, the above integral can be changed into:

$$O_{i1} \quad \frac{I}{4\pi R} \int_0^{d/2} \exp(\; 2 \quad {}_t Z) \frac{Z\,dZ}{\sqrt{R^2 \quad Z^2}} \tag{19}$$

Since the derived integral cannot be solved analytically, it should therefore be solved numerically. The following assumptions are considered for solving the integral:

– speed of neutrons v 10^7 m/s,
– observation time interval t 10^{-14} s,
– boundary layer thickness $d = v\ t$ 10^{-7} m,
– radius of the fuel element ≅ 0.5 cm, and
– total macroscopic cross section $_t$ 1500 cm^{-1}.

With the above assumptions, the derived integral is equal to:

$$O_{i1} \quad 0.1979\ 10^{\ 13} I \tag{20}$$

(2) The entrant neutrons, which participate in scattering reaction after entering the volume element and then escape from it. This group of neutrons should first have enough time to participate in a scattering reaction and then a fraction of them may escape from the volume element. Assume a neutron has traveled a distance x to participate in a scattering reaction and then travels the distance y, and subsequently escapes the region (fig. 5). It is clear that the sum of the distances x and y should satisfy the following condition:

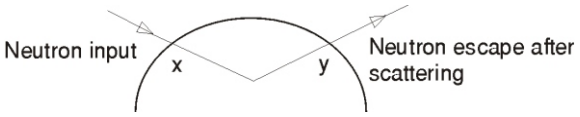

Figure 5. Escape probability of neutrons after scattering reaction

$$x \quad y \quad d \tag{21}$$

If the output term for this category of neutrons is shown by O_{i2}, then by hindsight and intuition one can conclude that O_{i2}, is much smaller than O_{i1}.

Here, one notes that the contribution of this group of neutrons to the output terms is very small. It is, therefore, to be deduced that this group of neutrons has only the opportunity to enter the element in the observation time interval t and that their later escape can be ignored. However, the final equation can be written as:

$$P \quad P_i \quad (A \quad A_i) \quad (O \quad O_{i1} \quad O_{i2}) \quad I \quad 0 \tag{22}$$

Simplification of the derived equation

As noted before, the boundary layer thickness is about 10^{-7} m. Therefore, the derived exponential sentences can be approximated using the Taylor expansion as follows:

$$\exp(\quad _t vt) \quad 1 \quad _t vt \tag{23}$$

Utilizing this approximation and applying it to the previously derived expressions we get:

$$P_{total} \quad P \quad P_i \quad vt\upsilon \quad _f \quad N \quad \frac{I}{2} \tag{24}$$

$$A_{total} \quad A \quad A_i \quad vt \quad _a \quad N \quad \frac{I}{2} \tag{25}$$

$$O_{total} \quad O \quad O_{i1} \quad O_{i2} \quad NPb(\upsilon \quad _f vt \quad 1 \quad _a vt)$$

$$0.1979\ 10^{\ 13} I \quad O_{i2} \tag{26}$$

and, the final equation may be written as:

$$N\ \upsilon \quad _f \quad _a \quad Pb\ \upsilon \quad _f \quad _a \quad \frac{1}{vt}$$

$$I\ \frac{\upsilon \quad _f \quad _a}{2} \quad \frac{0.1979\ 10^{\ 13}}{vt} \quad \frac{O_{i2}}{vt} \quad \frac{1}{vt} \quad 0 \tag{27}$$

In the above equation, υ and $_a$ can be ignored against the $1/vt$ term in the first parenthesis and likewise all of the terms in the second bracket against $1/vt$ term. With these approximations implemented, the final equation becomes:

$$N\ \upsilon \quad _f \quad _a \quad Pb\frac{1}{vt} \quad I\frac{1}{vt} \quad 0 \tag{28}$$

Consequently, the approximated and the simplified expressions for the production, absorption, input and output terms become:

$$P \quad N\upsilon \quad _f vt \tag{29}$$

$$A \quad N \quad _a vt \tag{30}$$

$$O \quad NPb \tag{31}$$

$$I \quad I \tag{32}$$

By recalling the expressions obtained for b and P and substituting them in Pb/vt we get:

$$\frac{Pb}{vt} \quad \frac{\frac{1}{4}\frac{2d}{R}}{vt} \quad \frac{1}{2R} \tag{33}$$

It is interesting to note that the dimension of the above term is the inverse of the unit of length and shall henceforth be defined as *leakage cross section* ($_L$). With this new nomenclature, the final equation becomes:

$$N(\upsilon \quad _f \quad _a \quad _L) \quad \frac{I}{vt} \quad 0 \tag{34}$$

There are few important observations worth noting in relation to the coefficient *b*. The first observation is that this coefficient is sensitive to the type of external boundary conditions applied, *i. e.* it depends on whether the net current equal to zero is used or the incoming current equal to zero is put. The other important point is that the DDM equations in static-state form for other geometries such as slab, sphere, square, triangular [7] are exactly identical except for their differences in the coefficient b, indicating its dependence on the geometry of the volume element.

Some notes on the input term (*I*)

As seen in fig. 6, the input from mesh volume i+1 to mesh volume i is the same as the output from mesh i+1 to mesh i. As a result, by using the output term which was calculated earlier, we get the following results:

$$I_{i\ 1,i} \quad O_{i\ 1,i} \quad N_{i\ 1} \quad _{L\,i\ 1,i}\, vt \qquad (35)$$

It is also known that $_{L\,i\ 1,i}$ is defined as:

$$_{L\,i\ 1,i} \quad \frac{Pb_{i\ 1}}{vt} \quad \frac{\frac{1}{4}\frac{2dR_i}{(R^2{}_{i\ 1} \quad R^2{}_i)}}{vt}$$

$$\frac{R_i}{2(R^2{}_{i\ 1} \quad R^2{}_i)} \qquad (36)$$

where, $b_{i\ 1}$ is the inner boundary layer fraction of the mesh interval i+1.

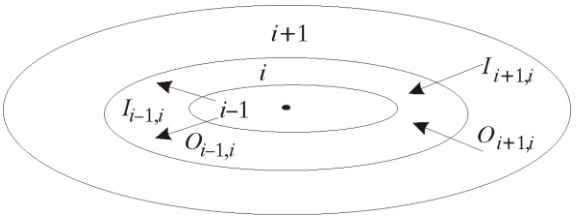

Figure 6. Input sentence calculation

Similarly, the input from mesh volume i – 1 to mesh volume i, as illustrated in fig. 6, is the same as the output from mesh i – 1 to mesh i. In likewise manner as before, we get:

$$I_{i\ 1,i} \quad O_{i\ 1,i} \quad N_{i\ 1} \quad _{L\,i\ 1,i}\, vt \qquad (37)$$

It is also known that $_{L\,i\ 1,i}$ is defined as:

$$_{L\,i\ 1,i} \quad \frac{Pb_{i\ 1}}{vt} \quad \frac{\frac{1}{4}\frac{2dR_{i\ 1}}{(R^2{}_{i\ 1} \quad R^2{}_{i\ 2})}}{vt}$$

$$\frac{R_{i\ 1}}{2(R^2{}_{i\ 1} \quad R^2{}_{i\ 2})} \qquad (38)$$

where $b_{i\ 1}$ is the outer boundary layer fraction of the mesh interval i – 1.

It is to be noted that the production, absorption, input and output terms are all stated in terms of the neutronic global variable *N*. Deriving the discrete equations for each of the mesh intervals and linking them together give rise to series of algebraic equations, which will have to be solved simultaneously. The derived matrix equation is AN = 0, where A is a n n coefficient matrix. N is the unknown n 1 matrix. Neutron population distribution and the eigenvalue and the corresponding multiplication factor *k* can all be obtained by solving the matrix equation.

IMPROVING THE DDM METHOD TO MULTIGROUP ENERGY

A cylindrical fuel element with volume V and surface S is assumed as a position element and a time interval t is selected, same as in one-group investigations. It is allowed that the neutron population depends on energy, but rather than treat the neutron energy variable E as a continuous variable, we will immediately discretize it into energy intervals or groups. The neutron energy range may be broken into G energy groups, as shown schematically in fig. 7. Notice that a backward indexing scheme was used for energy intervals. Due to this fact, that neutron usually loses energy during its lifetime; neutron up-scattering will be ignored in the process of neutron multigroup discrete equation production.

Neutrons in mesh interval i and energy group g are assumed and the ways of production and loss of them will be investigated.

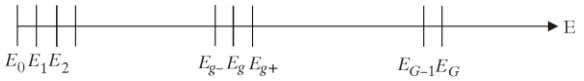

Figure 7. Energy discretizing

Production ways

I. Production from fission reaction may be shown as:

$$\chi_{ig}\,(\upsilon_{ig} N_{ig} V_{ig} \quad _{f\,ig})$$

$$\chi_{ig} \sum_{g\ 1}^{G} \upsilon_{ig} N_{ig} V_{ig} \quad _{f\,ig} \qquad (39)$$

N_{ig} is neutron population in mesh interval i with energy group g, and $V_{ig'}$ is the speed of neutrons in mesh interval i and energy group g'. The other parameters have the usual meaning in reactor physics.

II. Production of neutrons in mesh interval i and energy group g, due to the scattering of neutrons in other energy groups in the same mesh interval:

$$\sum_{g\ 1}^{g\ 1} N_{ig} V_{ig} \quad S\,ig \quad g \tag{40}$$

$S\,ig \quad g$ is neutron scattering cross section from energy group g to g for mesh interval i.

III. Production due to the escape of the neutrons in energy group g from mesh interval $i+1$ and $i-1$ to the desired mesh interval i:

$$N_{i\ 1,g} V_{i\ 1,g} \quad L\,i\ 1,i$$

$$N_{i\ 1,g} V_{i\ 1,g} \quad L\,i\ 1,i \tag{41}$$

It should be mentioned that leakage macroscopic cross section does not depend on the neutron energy and can be stated by the one-group energy theory as:

$$L\,i\ 1,i \quad \frac{R_{i\ 1}}{2(R_{i\ 1}^2 \quad R_{i\ 2}^2)}$$

and

$$L\,i\ 1,i \quad \frac{R_i}{2(R_{i\ 1}^2 \quad R_i^2)} \tag{42}$$

Loss ways

I. Loss due to absorption reaction may be shown as:

$$N_{ig} V_{ig} \quad a\,ig \tag{43}$$

II. Neutrons in mesh interval i and energy group g can escape to the other mesh intervals by the below rate:

$$N_{ig} V_{ig} \quad Li \tag{44}$$

Leakage macroscopic cross section Li from the mesh interval i can be stated by the one-group energy theory as:

$$Li \quad \frac{1}{2(R_i \quad R_{i\ 1})} \tag{45}$$

III. Neutrons in mesh interval i and energy group g, can participate in the scattering reaction and therefore exit from the energy group g by the below rate:

$$\sum_{g\ g}^{g\ g} N_{ig} V_{ig} \quad S\,i,g \quad g \quad \sum_{g\ g}^{g\ g} N_{i,g} V_{i,g} \quad S\,i,g \quad g \tag{46}$$

$S,ig \quad g$ may be shown by S,ig and named total scattering cross section for neutrons in mesh interval i and energy group g that transfers the neutrons from energy group g to the other energy groups. Therefore, the final neutron discrete equation in multigroup energy for mesh interval i and energy group g, for a cylindrical geometry becomes:

$$N_{i\ 1,g} V_{i\ 1,g} \quad L\,i\ 1,i \quad V_{i,g} N_{i,g} (\ a\,i,g \quad S\,i,g \quad Li)$$

$$N_{i\ 1,g} V_{i\ 1,g} \quad L\,i\ 1,i \quad \sum_{g\ g} S\,i,g \quad g\, N_{i,g} V_{i,g}$$

$$\frac{1}{k} \chi_{i,g} (\sum_{g\ 1}^{G} \upsilon_{i,g} N_{i,g} V_{i,g} \quad f\,i,g) \tag{47}$$

where k is the multiplication factor of the desired medium. The derived discrete equation can be written as a matrix form $AN = (1/k)\,BN$ where A and B are $[(n \quad g) \quad (n \quad g)]$ coefficient matrices. N is the unknown $[(n \quad g) \quad 1]$ matrix.

RESULTS AND DISCUSSION

To evaluate the DDM method in one-group energy, two typical problems have been solved using the following data (see tab. 1). First, a fuel element made up of uranium-235 with 1 cm radius is considered. Next, a fuel element with the associated clad and coolant regions is considered. The clad and the coolant thicknesses are taken as 0.1 cm and 0.3 cm, respectively. The type of material assumed for the clad is Zr and that of the coolant is H_2O. These examples are solved for two widsespread external boundary conditions, namely $J_- = 0$ and $J_{net} = 0$. The same problems have also been solved with the MCNP-4B [9] and the ANISN codes [10]. Figures 8, 9, and 10 show the results for comparison.

Table 1. Data used in the one-group energy test examples 7

Elements	υ	$_a$ cm^{-1}	$_f$ cm^{-1}	$_s$ cm^{-1}
U-235	2.5	3.3+1[a]	2.8+1	4.81–1
Zr	0.00	7.7–3	0.00	3.03–1
H_2O	0.00	2.26–2	0.00	2.069

[a] read as 3.3 10^1

To evaluate the validity of the DDM method in multigroup energy, two criticality search problems have been solved in two-group energy. NJOY-97 [11] has been applied to extract required data from ENDF/B-VI [12] in two-group energy. The generated data for required elements are presented in tabs. 2 and 3. Using the produced two-group energy libraries, first a fuel element made up of uranium-235 with 4.8 cm radius (critical radius) is

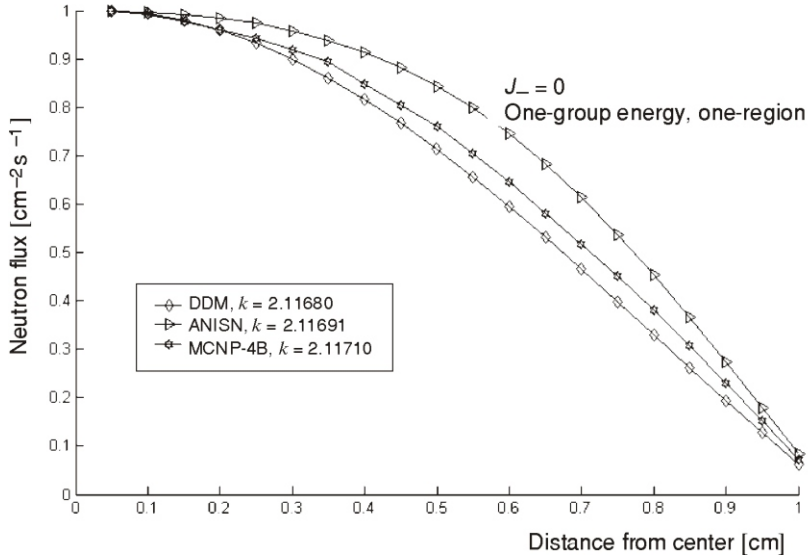

Figure 8. Normalized neutron flux versus distance for one fuel element with radius 1 cm; one-group theory (input current into the last mesh is zero)

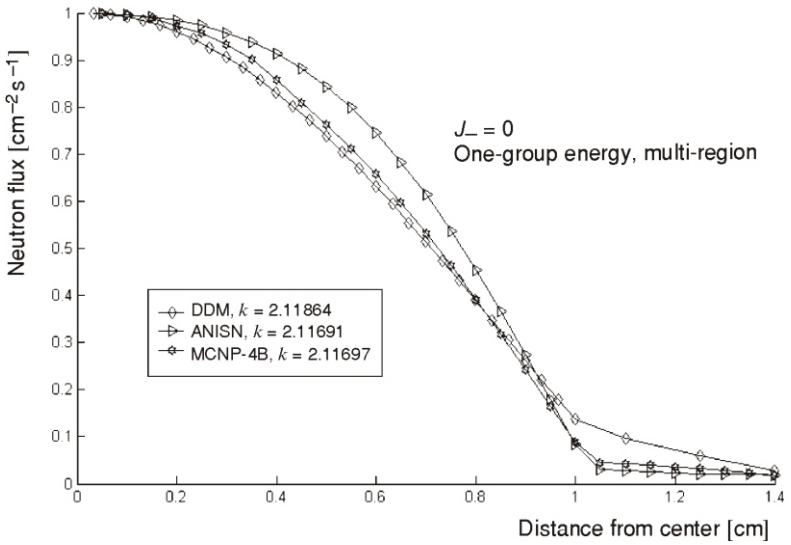

Figure 9. Normalized neutron flux versus distance for one fuel element associated with clad and coolant; one-group theory (input current into the last mesh is zero)

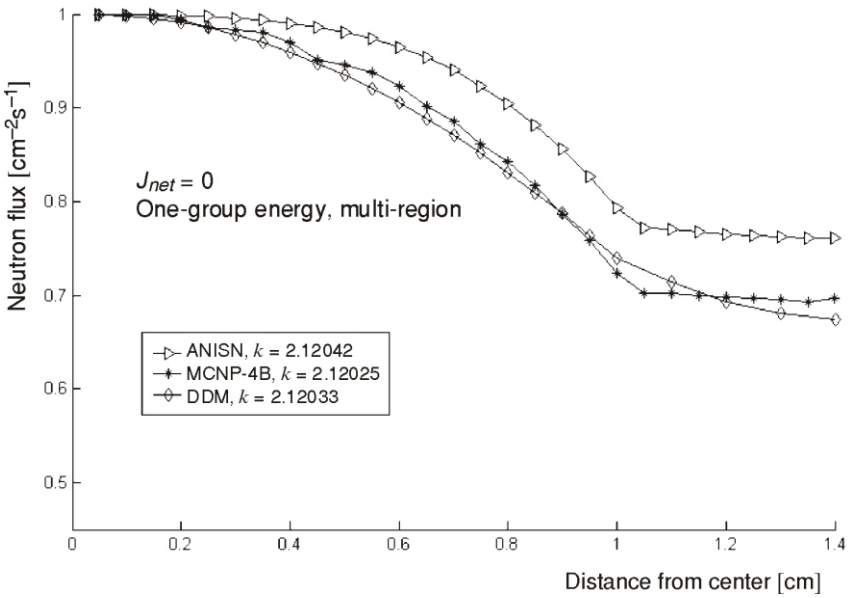

Figure 10. Normalized neutron flux versus distance for one fuel element associated with clad and coolant; one-group theory (net current into the last mesh is zero)

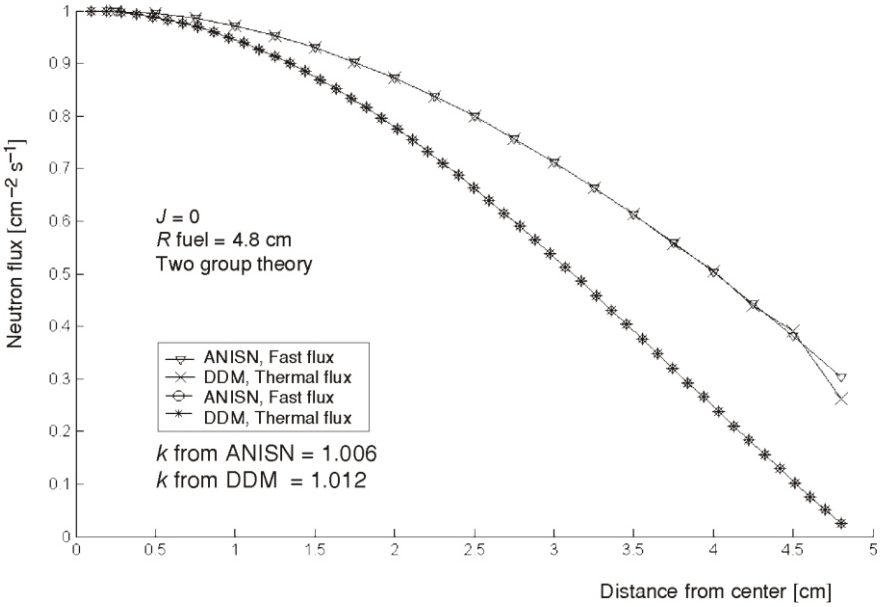

Figure 11. Normalized neutron flux versus distance for one fuel element with critical radius; two-group theory (input current into the last mesh iz zero)

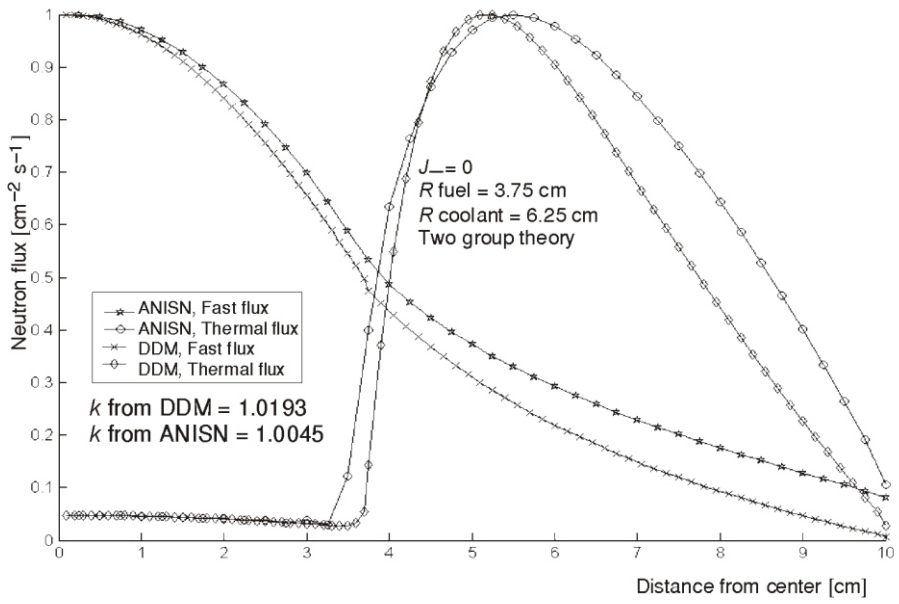

Figure 12. Normalized neutron flux versus distance for a fuel element surrounded by coolant with critical radius; two-group theory (input current into the last mesh is zero)

Table 2. The fast data used in two-group energy test problems

Elements	Σ_{a1} 10^{-28} m^2]	$_{f1}$ 10^{-28} m^2	$_{S1}$ 10^{-28} m^2	$_{S1}$ 10^{-28} m^2	$_{S1\ 2}$ 10^{-28} m^2	υ
U-235	1.1865	1.1845	6.386	6.460	1.2–9	2.85
H-1	3.63–5	00.00	2.534	2.534	1.3–6	0.00
O-16	4.00–2	00.00	2.245	2.245	1.3–15	0.00

Table 3. The thermal data used in two-group energy test problems

Elements	Σ_{a2} 10^{-28} m^2	$_{f2}$ 10^{-28} m^2	$_{S2}$ 10^{-28} m^2	$_{S2\ 2}$ 10^{-28} m^2	$_{S2\ 1}$ 10^{-28} m^2	υ
U-235	119.19	102.68	14.287	14.287	00.00	2.41
H-1	7.96–2	00.00	21.236	21.236	00.00	00.00
O-16	4.57–5	00.00	3.898	3.898	00.00	00.00

considered. Next, a fuel element with the associated coolant region is considered. The critical radius of the fuel element which is surrounded by 6.25 cm coolant, changes to 3.75 cm, for the external boundary condition, $J_- = 0$. The same problems have also been solved with ANISN code. Figures 11 and 12 show the results for comparison.

It should be noticed that the neutron fluxes were normalized between 0 and 1. In reality, the thermal flux is of the order 10^{-14} in comparison to the order 1 of fast flux.

CONCLUSION

The DDM method is very simple to set up and it obviates the need to go through the differential formulation process first. DDM starts from the basic and fundamental meanings of neutron physics, then passes to the desired meshing scheme of the geometry on hand, and, by writing the balance equa-

tion for each mesh interval and combining them, we are finally lead to the sought algebraic matrix equation. This method in one-group and multigroup energy produces reasonable results, which are comparable with those obtained from the MCNP-4B and the ANISN codes.

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ДИРЕКТНА ДИСКРЕТНА МЕТОДА И ЊЕНА ПРИМЕНА У ТРАНСПОРТУ НЕУТРОНА

У раду је приказана једна нова метода за неутронске прорачуне. Овај поступак, назван директна дискретна метода, једноставнији је од уобичајене примене неутронске транспортне једначине, и у већој је сагласности са физичким значењима задатка. Метода полази од формирања мреже разматране геометрије засноване на физици проблема и образовању једначина баланса за сваки интервал мреже. Повезујући мрежне интервале, коначан систем дискретних једначина непосредно се добија без потребе да се претходно успостави диференцијална транспортна једначина неутрона. У раду су образоване једногрупне и мултигрупне дискретне транспортне једначине неутрона за горивни елемент цилиндричног облика са и без придружене кошуљице и зоне хладиоца – свака са два различита услова на спољашњој граници. Ваљаност резултата добијених новом методом проверена је поређењем са резултатима оствареним програмима MCNP-4B и ANISN.